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ULTRASOUND NOTES

FRAME RATES

Time for each line is $\frac{2r_{max}}{c}$

e.g. $r_{max} = 20$ cm.

$$T = \frac{2\,(20\text{ cm})}{1500\text{ m/s}} = \frac{.04}{1500}\text{ s} = 267\ \mu\text{sec}$$

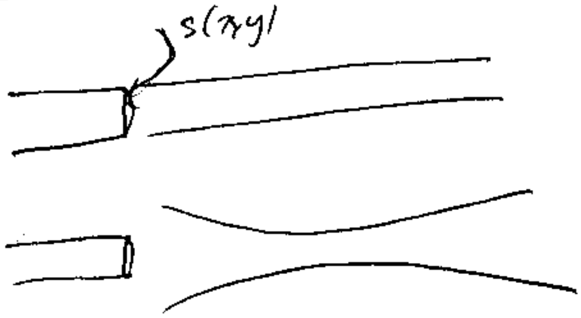
for 1 mm lateral resolution

$$N_{lines} \approx \frac{20}{0.1} = 200$$

Total time is $N_{lines} \cdot T = .0534$ seconds

≈ 19 frames/second

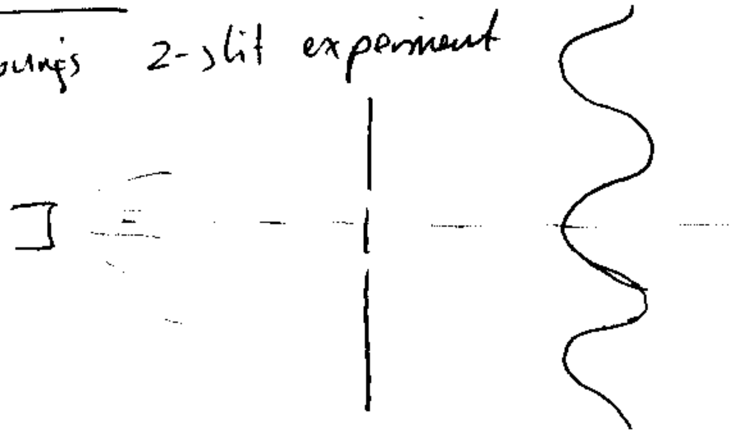
Lateral Resolution

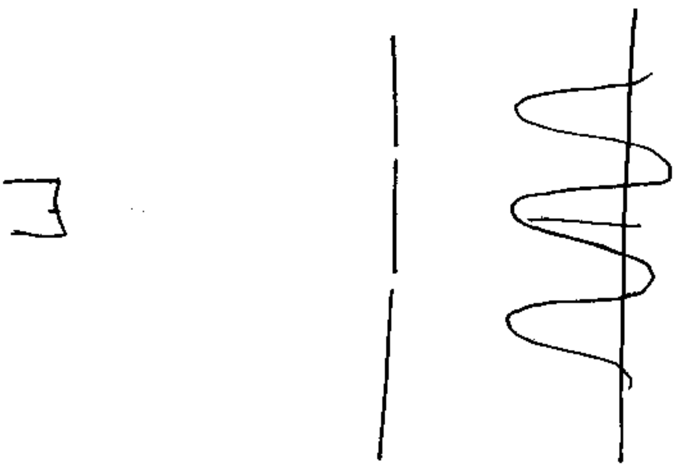


Question: How to achieve a narrow beam?
How to focus acoustic energy?

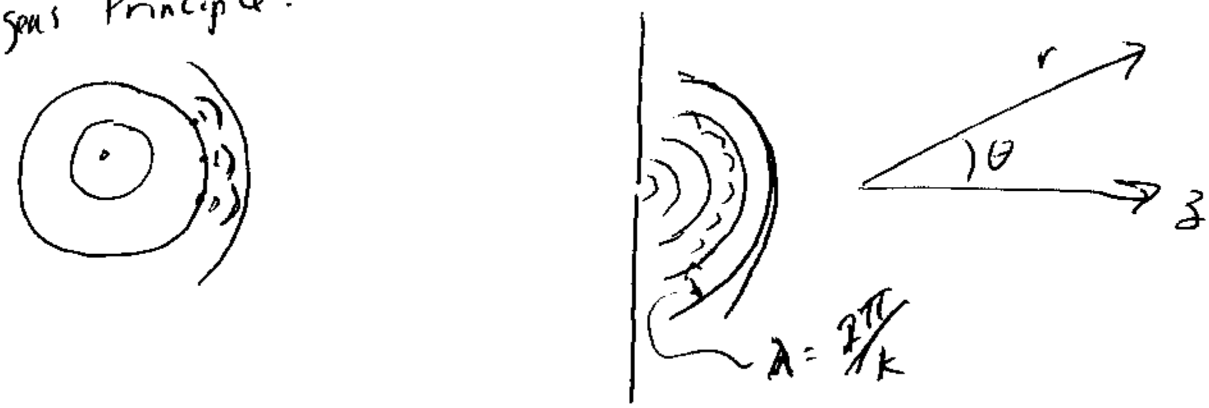
Diffraction

Young's 2-slit experiment



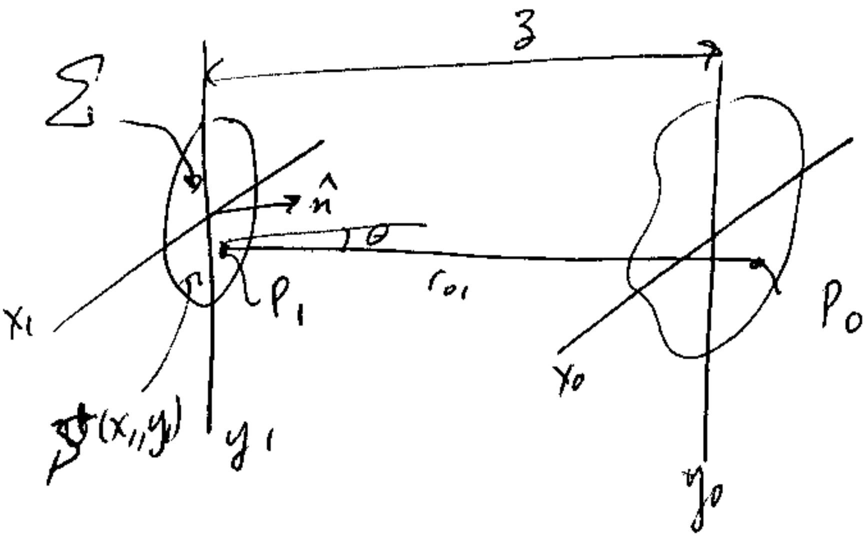


Huygens Principle:



Pressure wave due to a point source is

$$p(r) = \frac{e^{j\overset{\text{wavelength}}{k}r}}{r} \underbrace{\cos\theta}_{\text{obliquity factor}}$$



$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$U(P_0) = \iint_{\Sigma} h(P_0, P_1)\, U(P_1)\, dA$$

$$= \iint_{\Sigma} h(x_0, y_0; x_1, y_1)\, U(x_1, y_1)\, \cancel{dA}\, dx_1\, dy_1$$

where $h(x_0, y_0; x_1, y_1)$ is the impulse response

$$= \frac{1}{j\lambda} \frac{\exp(jkr_{01})}{r_{01}} \cos\theta$$

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small angle approximation (paraxial approximation)

$$\cos\theta \approx 1$$
$$r_{01} \approx z$$
$$h \approx \frac{1}{j\lambda z} \exp(jkr_{01})$$

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Fresnel Approximation (near-field)

$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$= z\sqrt{1 + \left(\frac{x_1 - x_0}{z}\right)^2 + \left(\frac{y_1 - y_0}{z}\right)^2}$$

$$\approx z\left[1 + \frac{1}{2}\left(\frac{x_1 - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y_1 - y_0}{z}\right)^2\right]$$

$$h \approx \frac{\exp(jkz)}{j\lambda z}\exp\left[\frac{jk}{2z}\left[(x_1 - x_0)^2 + (y_1 - y_0)^2\right]\right]$$

- We have approximated the spherical wavefronts with quadratic wave fronts.

- Most ultrasound transducers operate in the Fresnel zone.

$$U(P_0) = \iint \frac{\exp(jkz)}{j\lambda z}\exp\left[\frac{jk}{2z}\left[(x_1 - x_0)^2 + (y_1 - y_0)^2\right]\right] dx_1 dy_1$$
$$\cdot S(x_1, y_1)$$

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z}\left[S(x_0, y_0) ** \exp\left(\frac{jk}{2z}\left(x_0^2 + y_0^2\right)\right)\right]$$

Fraunhofer Approximation (FAR-FIELD)

$$r_{01} \approx z\left[1 + \frac{1}{2}\left(\frac{x_1 - x_0}{z}\right)^2 + \frac{1}{2}\left(\frac{y_1 - y_0}{z}\right)^2\right]$$

$$= z\left[1 + \frac{1}{2z^2}\left(x_1^2 - 2x_1x_0 + x_0^2\right) + \frac{1}{2z^2}\left(y_1^2 - 2y_1y_0 + y_0^2\right)\right]$$

$$= z + \frac{1}{2z}\left(x_0^2 + y_0^2\right) + \frac{1}{2z}\left(x_1^2 + y_1^2\right) - \frac{1}{z}\left(x_1x_0 + y_1y_0\right)$$

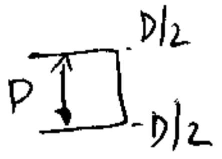
phase term is $k \cdot r_{01}$

consider the term $\frac{k}{2z}\left(x_1^2 + y_1^2\right)$ ← phase due to where we are on the transducer

We want this term to be much less 1 radian

$$\frac{k}{2z}\left(x_1^2 + y_1^2\right) \ll 1$$

$$\boxed{z \gg \frac{k}{2}\left(x_1^2 + y_1^2\right) = \frac{\pi\left(x_1^2 + y_1^2\right)}{\lambda}}$$



$$\boxed{z \gg \frac{\pi D^2}{4\lambda}} \approx \frac{D^2}{\lambda}$$

Fraunhofer Approximation Continued

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$$\exp(jkr_{01}) \approx \exp(jkz)\exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right) \cdot \exp\left(-j\frac{k}{z}(x_0x_1+y_0y_1)\right)$$

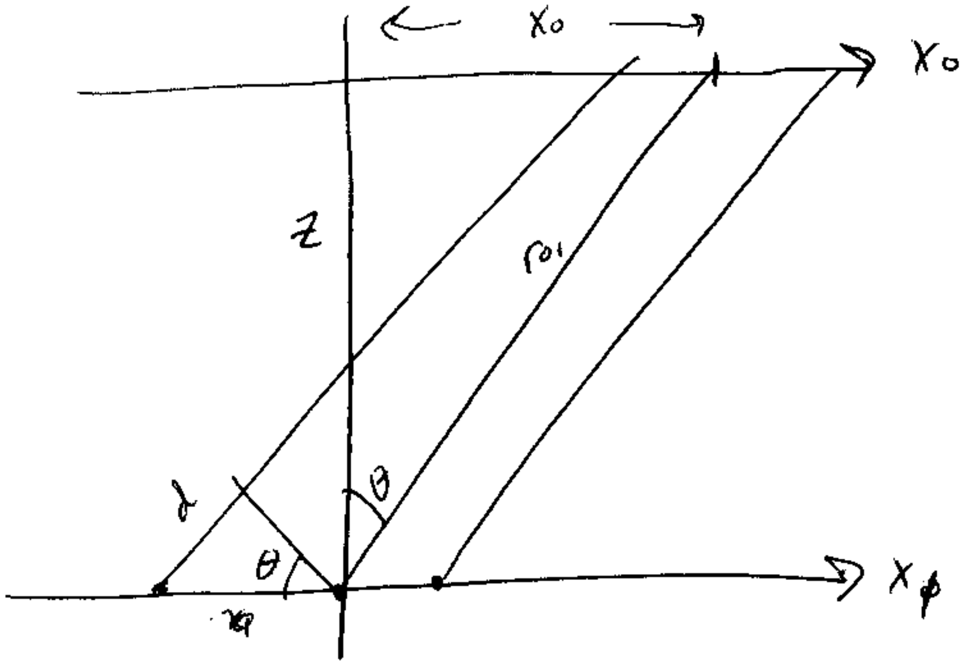
$$U(x_0,y_0) = \left(\frac{\exp(jkz)}{j\lambda z}\cdot \exp\left(\frac{jk}{2z}(x_0^2+y_0^2)\right)\right)\iint \cancel{\phi}(x_1,y_1)\exp\left[-j\frac{k}{z}(x_0x_1+y_0y_1)\right]dx_1dy_1$$

call this $\dfrac{\exp(j\nu)}{j\lambda z}$

$$= \frac{\exp(j\nu)}{j\lambda z}\iint \cancel{\phi}(x_1,y_1)\exp\left[-j\left(\frac{2\pi}{\lambda z}x_0x_1+\frac{2\pi}{\lambda z}y_0y_1\right)\right]dx_1dy_1$$

$$= \frac{\exp(j\nu)}{j\lambda z}\mathcal{F}\left[\cancel{\phi}(x,y)\right]\Bigg|_{k_x=\frac{x_0}{\lambda z},\ k_y=\frac{y_0}{\lambda z}}$$

Fraunhofer (plane-wave interpretation).



$$d = x_1 \sin\theta \qquad \sin\theta = \frac{x_0}{r_{01}} \approx \frac{x_0}{z}$$

So
$$d = \frac{x_1 x_0}{z}$$

relative phase is
$$-\frac{2\pi d}{\lambda} = -\frac{2\pi x_1 x_0}{\lambda z}$$

$$U(x_0) \approx \int \mathcal{U}(x_1)\, e^{-j 2\pi x_1 x_0 / \lambda z}\, dx_1$$

$$= F(\mathcal{U}(x))\Big|_{k_x = \frac{x_0}{\lambda z}}$$

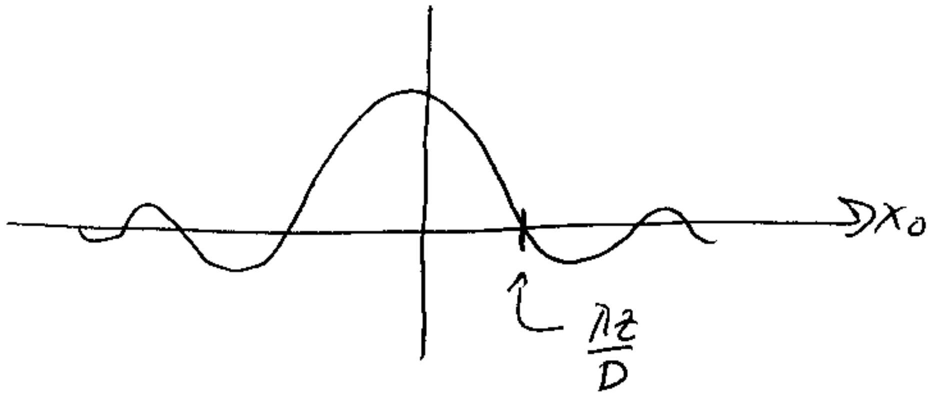
Example :

let $s(x,y) = \mathrm{rect}\left(\frac{x}{D}\right)\mathrm{rect}\left(\frac{y}{D}\right)$

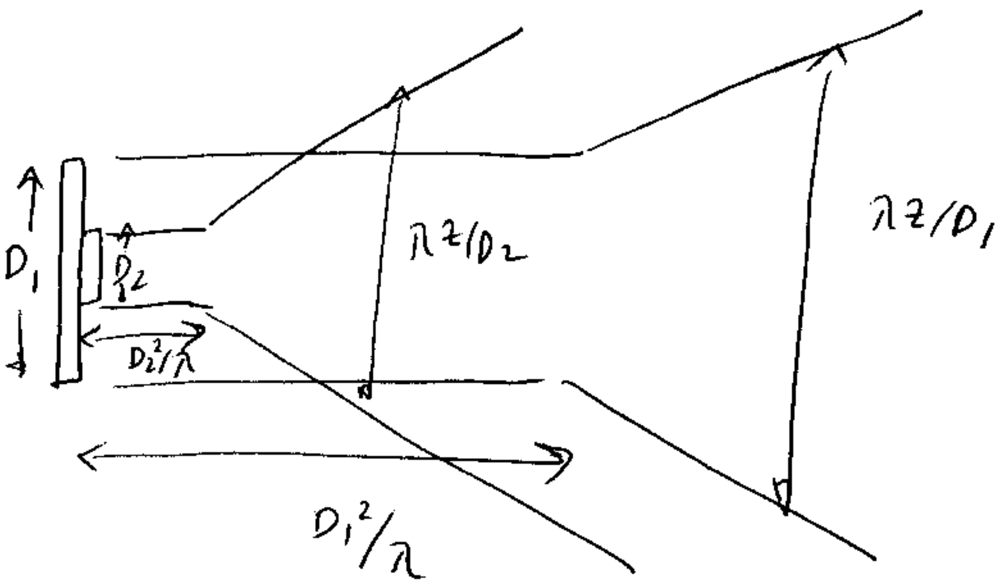
$$U(x_0,y_0) = \frac{\exp(jV)}{z} \cdot \mathcal{F}\left[s(x,y)\right]\Big|_{k_x=\frac{x_0}{\lambda z},\ k_y=\frac{y_0}{\lambda z}}$$

$$= \exp\left(\frac{jV}{z}\right) D^2 \,\mathrm{sinc}(Dk_x)\,\mathrm{sinc}(Dk_y)$$

$$= \exp\left(\frac{jV}{z}\right) D^2 \,\mathrm{sinc}\left(\frac{Dx_0}{\lambda z}\right)\mathrm{sinc}\left(\frac{Dy_0}{\lambda z}\right)$$



thus beam width in far field is $\propto \frac{\lambda z}{D}$



Choice of transducer Dimensions

near field region $z < D^2/\lambda$

$D_{opt} \hat{=} \sqrt{\lambda z_{max}}$.

$z_{max} = 20$ cm.

$\lambda = 0.5$ mm (corresponds to $f = \frac{c}{\lambda} = 3$ MHz)

$D_{opt} = 1.0$ cm

Fresnel Zone Revisited

Can write as

$$U = \frac{\exp(j\nu)}{j\lambda z} \iint s(x_1, y_1) \exp\left(j\frac{k}{2z}(x_1^2 + y_1^2)\right) \exp\left(-j\left(\frac{2\pi}{\lambda z} x_0 x_1 + \frac{2\pi}{\lambda z} y_0 y_1\right)\right) dx_1 dy_1$$

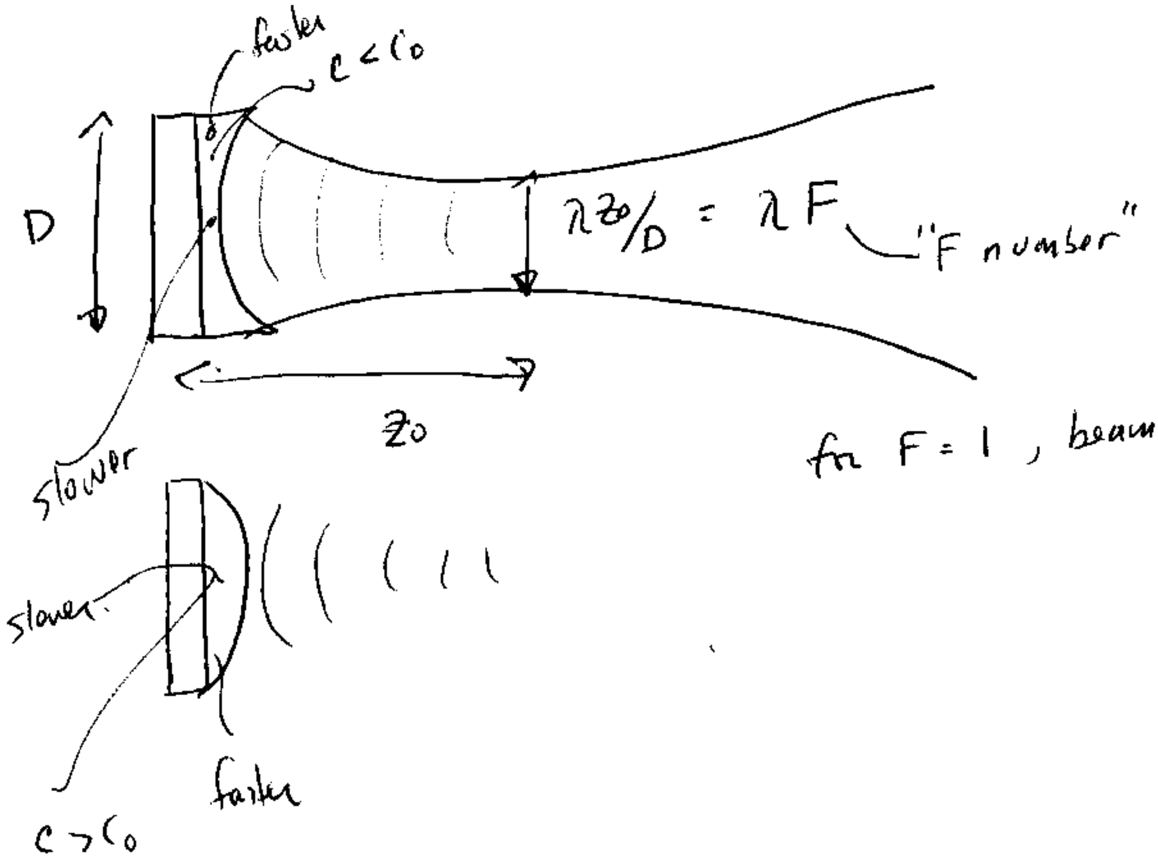
$$= \frac{\exp(j\nu)}{j\lambda z} \mathcal{F}\Big[\underbrace{s(x,y) \exp\left(j\frac{k}{2z}(x_1^2 + y_1^2)\right)}_{S_{eff}(x,y)} \Big]$$

(Focusing)

make $s(x,y) = s_0(x,y) \exp\left(-j\frac{k}{2z_0}(x_1^2+y_1^2)\right)$

then at z_0 $\quad v = \dfrac{\exp(j\cdot)}{j\lambda z}\,\mathcal{F}\left[s_0(x,y)\right]$

therefore at focal depth
beam width $\approx \dfrac{\lambda z_0}{D}$



for $F = 1$, beamwidth $= \lambda$

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BEAM steering

let $s(x,y) = s_0(x,y)\, e^{-j\frac{k}{2z_0}(x_1^2 + y_1^2)}\, e^{j2\pi \frac{x'}{\lambda_0 z} x}$

then

~~$\mathcal{F}(s(x,y))$~~

$$U = \mathcal{F}\left[s_0(x,y)\, \underbrace{e^{j2\pi x' \frac{x}{\lambda_0 z}}} \right]\Bigg|_{k_x = \frac{x_0}{\lambda_0 z} \;\; k_y = \frac{y_0}{\lambda_0 z}}$$

modulation by $e^{j2\pi k_x x}$

$$= S(k_x, k_y) * \delta(k_x - k_x')$$

$$= S(k_x - k_x', k_y)$$

$$= S\left(\frac{x_0 - x'}{\lambda_0 z}, \frac{y_0}{\lambda_0 z} \right)$$

shifts beam by x'