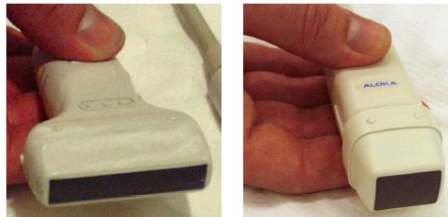


Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
Ultrasound Lecture 1

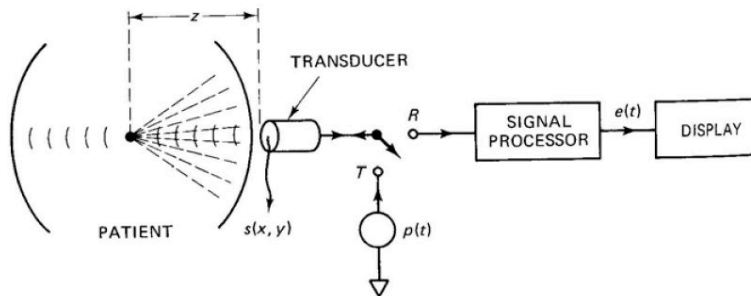
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Basic System

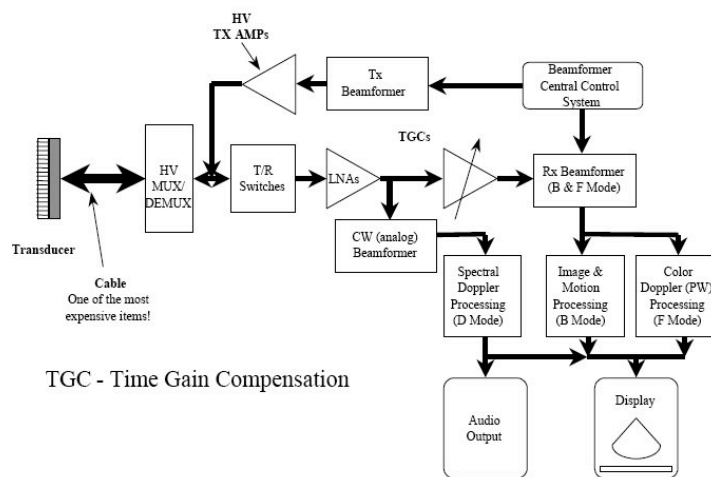


Echo occurs at $t=2z/c$ where c is approximately 1500 m/s or 1.5 mm/ μ s

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Macovski 1983

Basic System



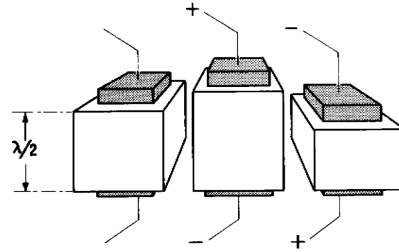
TGC - Time Gain Compensation

Brunner 2002

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Macovski 1983

Transducer

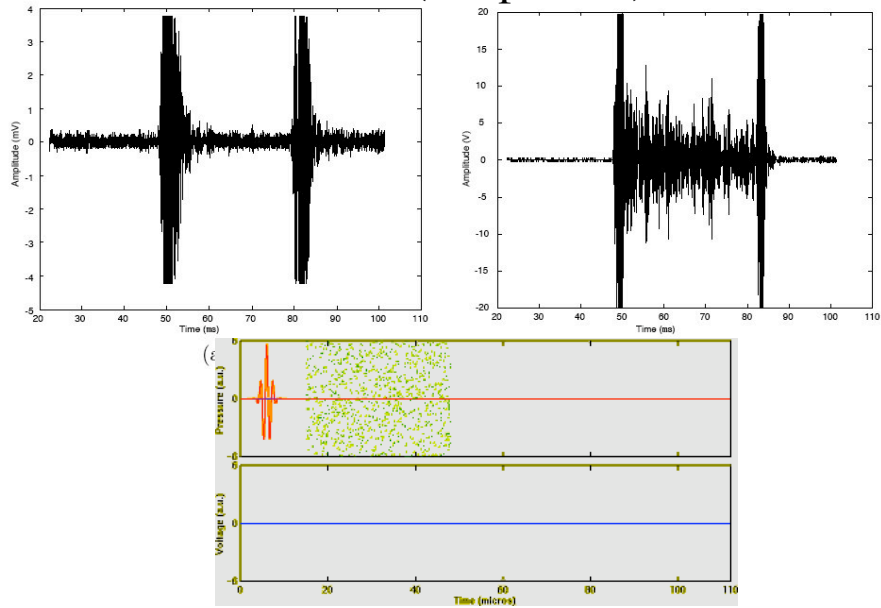


Seutens 2002

www.engineering.uiowa.edu/~bme_285/Lecture/Notes%20on%20Ultrasound.pdf

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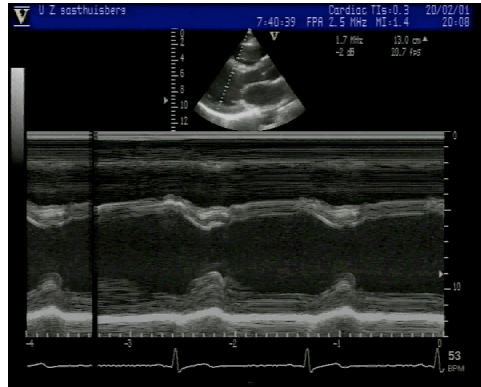
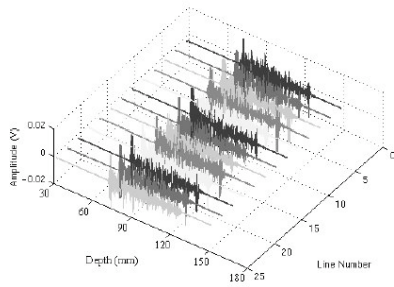
A-Mode (Amplitude)



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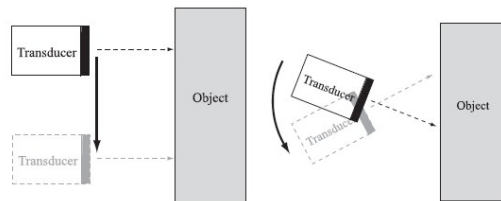
M-Mode (Motion)



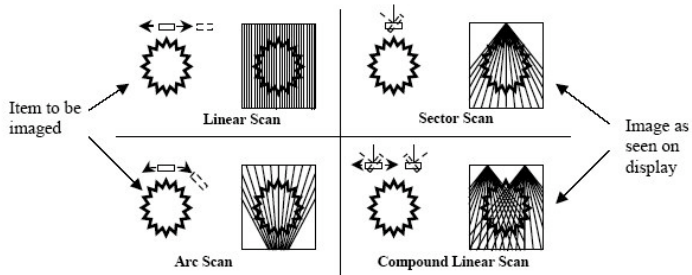
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B-Mode (Brightness)



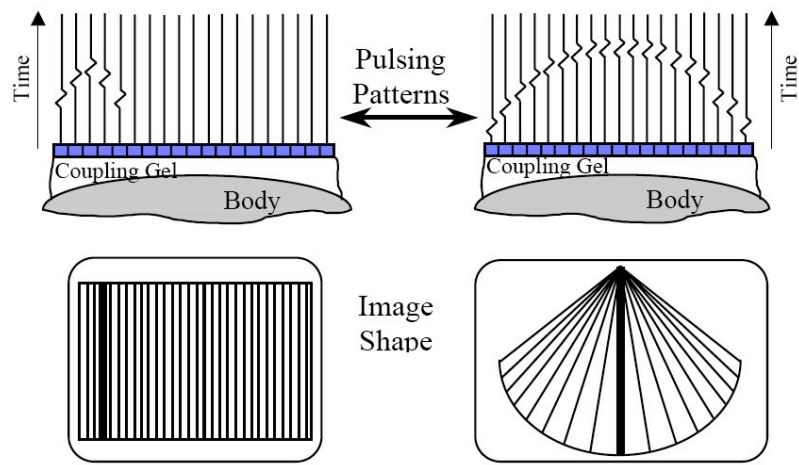
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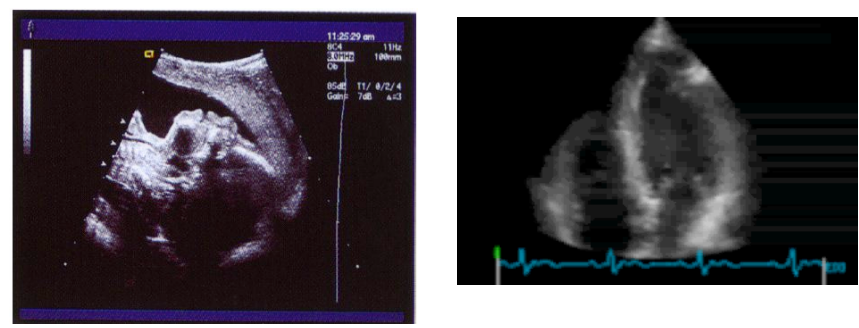
B-Mode (Brightness)



Brunner 2002

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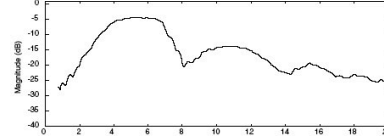
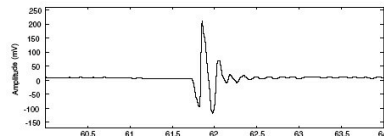
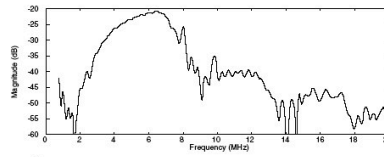
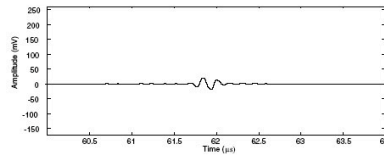
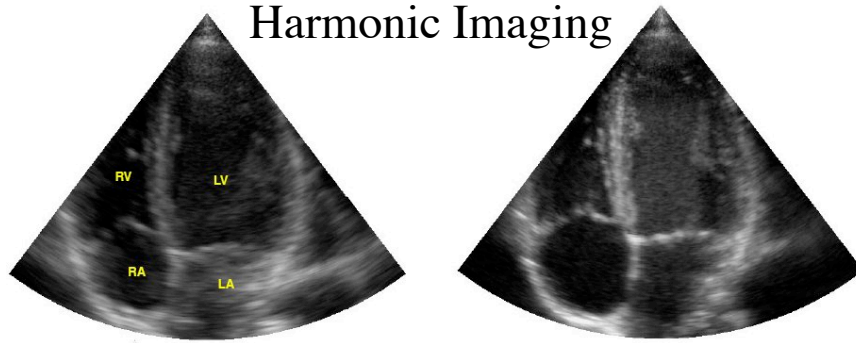
B-Mode



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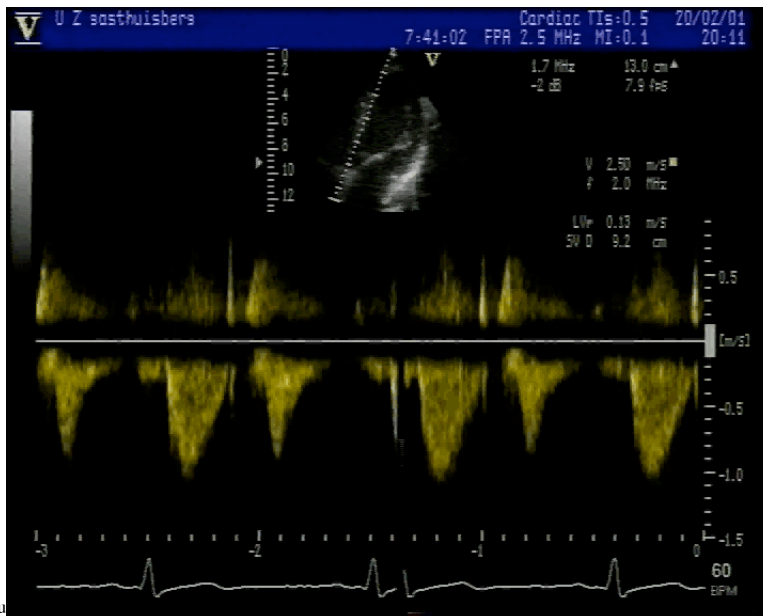
Harmonic Imaging



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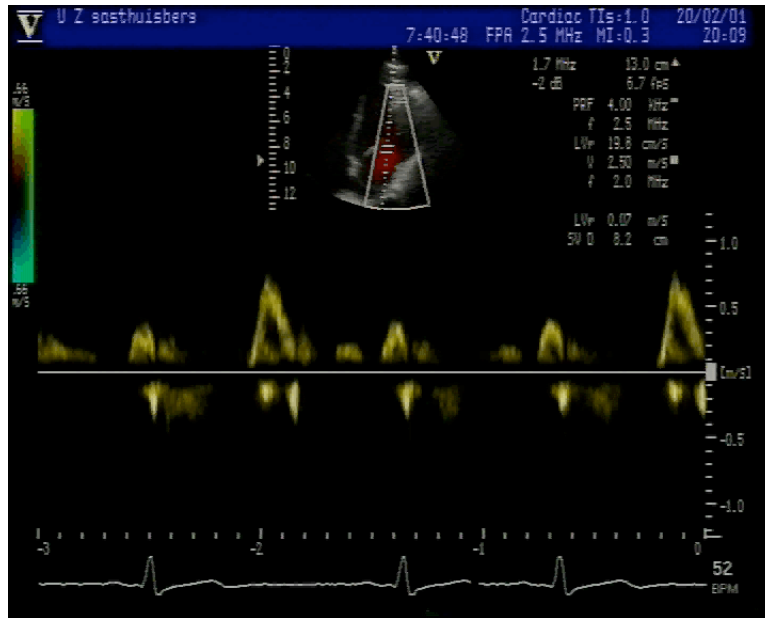
CW Doppler Imaging



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Seutens 2002

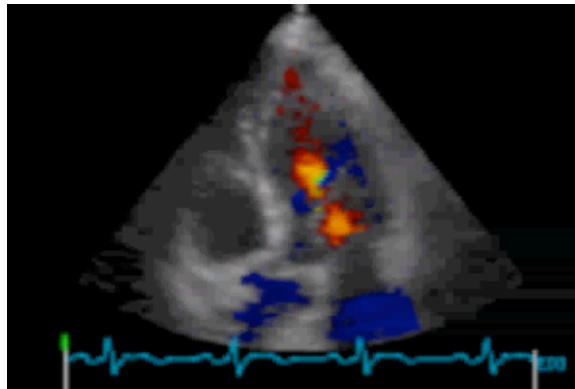
PW Doppler Imaging



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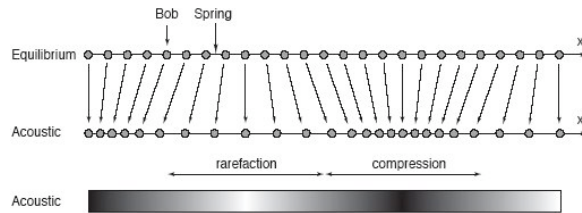
Color Doppler Imaging



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Acoustic Waves



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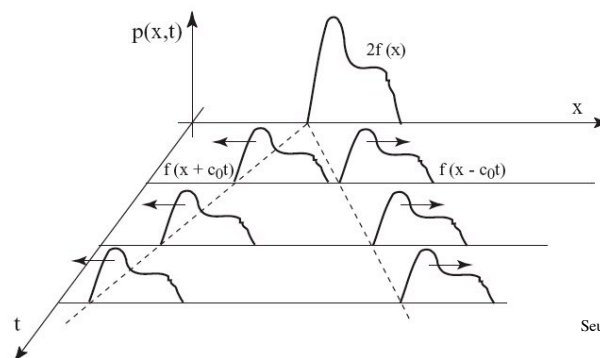
Suetens 2002

Acoustic Wave Equation

$$\nabla^2 p = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

Solutions are of the form

$$p(x, t) = A_1 f_1(x - ct) + A_2 f_2(x + ct)$$



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Suetens 2002

Acoustic Wave Equation

Solutions of the wave equation

Plane wave

$$p(z,t) = \exp(j2\pi f(t - z/c))$$

Superposition of plane waves

$$p(z,t) = \int_{-\infty}^{\infty} P(f) \exp(j2\pi f(t - z/c)) df$$

$$\text{At } z = 0: p(0,t) \equiv p(t) = \int_{-\infty}^{\infty} P(f) \exp(j2\pi ft) df = F^{-1}(P(f))$$

$$p(z,t) = p(0,t - z/c) = p(t - z/c)$$

Spherical Wave

$$p(r,t) = \frac{1}{r} \exp(j2\pi f(t - r/c))$$

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Impedance

$$\text{Impedance } Z = \frac{\text{Pressure}}{\text{Velocity}} = \frac{P}{v} = \rho c$$

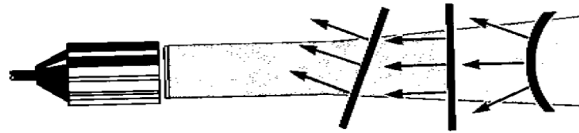
density kg/m³

speed of sound
Brain 1541 m/s
Liver 1549
Skull bone 4080 m/s
Water 1480 m/s

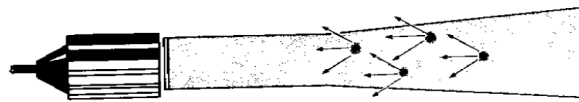
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Echos

SPECULAR ECHOES

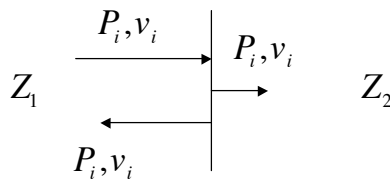


SCATTERED ECHOES



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Specular Reflection



Material	Reflectivity
Brain-skull	0.66
Fat-muscle	0.10
Muscle-blood	0.03
Soft-tissue-air	.9995

$$v_i - v_r = v_t \quad (\text{velocity boundary condition})$$

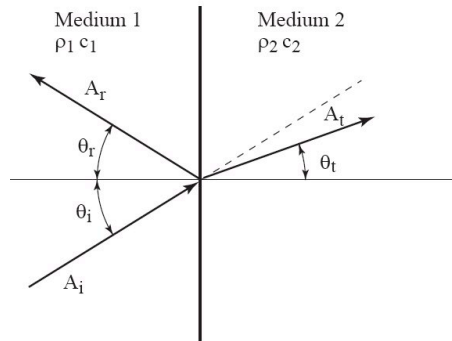
$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_t}{Z_2}$$

$$P_i + P_r = P_t \quad (\text{pressure boundary condition})$$

$$R = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx \frac{\Delta Z}{Z_0}$$

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Reflection and Refraction



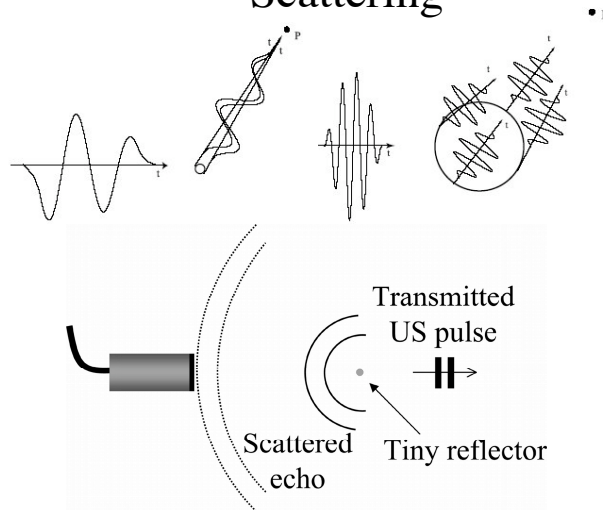
$$\frac{\sin \theta_i}{c_1} = \frac{\sin \theta_r}{c_1} = \frac{\sin \theta_t}{c_2}$$

$$R = \frac{P_r}{P_i} = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

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Scattering



Point scatterers retransmit the incident wave equally in all direction (e.g. isotropic scattering).

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Attenuation

Loss of acoustic energy during propagation.
Conversion of acoustic energy into heat.

$$H(f, z) = \exp(-\alpha(f)z) \approx \exp(-\alpha_0 f^n z)$$

For frequencies used in medical ultrasound, $n \approx 1$.

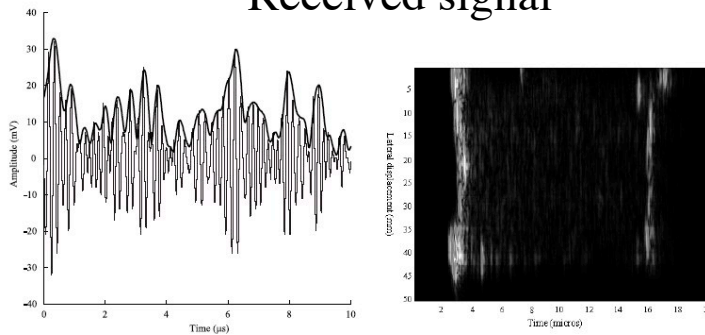
In liver, typical value is $\alpha_0 = 0.5$ db/cm/MHz
For tissues in general, $\alpha_0 \approx 1.0$ db/cm/MHz

Example: Liver at 2 MHz, attenuation 1 dB/cm.
After the 6 cm, 6 dB of attenuation.
 $10^{(-3/20)} = 0.5$

$$\text{Recall dB} \equiv 20 \log_{10} (A_z / A_0)$$

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Received signal



$$e(t) = K \int \int \int \frac{e^{-2\alpha z}}{z} R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

Attenuation

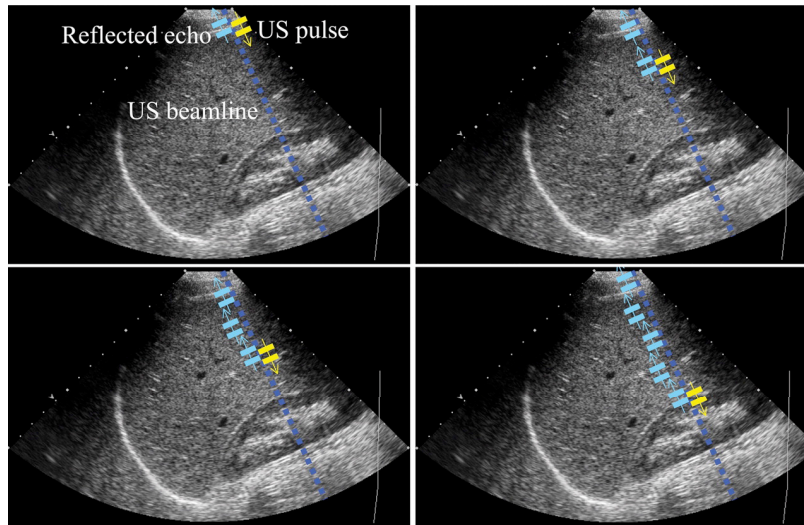
Reflection/Scattering

Beam width

Pulse

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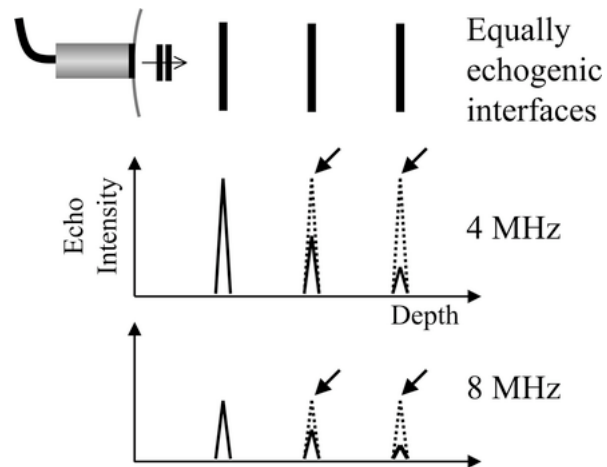
Received signal



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<http://radiographics.rsnajnl.org/content/vol23/issue4/images/large/g03jl25c1x.jpeg>

Attenuation Correction



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Attenuation Correction and Signal Equation

$$e(t) = K \int \int \int \frac{e^{-2\alpha z}}{z} R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

$$\approx K \frac{e^{-\alpha ct}}{ct/2} \int \int \int R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

$$e_c(t) = cte^{\alpha ct} e(t)$$

$$\approx K \int \int \int R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

$$= \frac{c}{2} \int \int \int R(x, y, c\tau/2) s(x, y) p(t - \tau) dx dy d\tau$$

$$= K \frac{c}{2} \left[R(x, y, ct/2) *** s(-x, -y) p(t) \right] \Big|_{x=0, y=0}$$

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Signal Equation Example

$$e_c(t) \approx K \int \int \int R(x, y, z) s(x, y) p(t - 2z/c) dx dy dz$$

$$= K \frac{c}{2} \left[R(x, y, ct/2) *** s(-x, -y) p(t) \right] \Big|_{x=0, y=0}$$

Let $R(x, y, z) = \delta(x)\delta(y)\delta(z - z_0) + \delta(x)\delta(y)\delta(z - z_1)$
and $s(x, y) = \text{rect}(x/L)\text{rect}(y/L)$

$$e_c(t) = K \int \int \int [\delta(x)\delta(y)\delta(z - z_0) + \delta(x)\delta(y)\delta(z - z_1)] \text{rect}(x/L)\text{rect}(y/L) p(t - 2z/c) dx dy dz$$

$$= K [p(t - 2z_0/c) + p(t - 2z_1/c)]$$

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Signal Equation Example

$$e_c(t) \approx K \int \int \int R(x,y,z) s(x,y) p(t-2z/c) dx dy dz$$

$$= K \left[R(x,y,ct/2) *** s(-x,-y) p(t) \right] \Big|_{x=0,y=0}$$

Let $R(x,y,z) = \delta(x)\delta(y)\delta(z-z_0) + \delta(x)\delta(y)\delta(z-z_1)$

What happens to the signal as we move the transducer to some arbitrary position x_0, y_0 ?

$$e_c(t, x_0, y_0) = K \int \int \int [\delta(x)\delta(y)\delta(z-z_0) + \delta(x)\delta(y)\delta(z-z_1)] s(x-x_0, y-y_0) p(t-2z/c) dx dy dz$$

$$= K [p(t-2z_0/c) s(-x_0, -y_0) + p(t-2z_1/c) s(-x_0, -y_0)]$$

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Signal Equation Summary

In general, we can write

$$e_c(t, x_0, y_0) = K \int \int \int R(x,y,z) s(x-x_0, y-y_0) p(t-2z/c) dx dy dz$$

$$= K \frac{c}{2} \left[R(x,y,ct/2) *** s(-x,-y) p(t) \right] \Big|_{x=x_0, y=y_0}$$

$$e_c(z', x_0, y_0) = K \int \int \int R(x,y,z) s(x-x_0, y-y_0) p(2(z'-z)/c) dx dy dz$$

$$= \left[R(x,y,z') *** s(-x,-y) p(2z'/c) \right] \Big|_{x=x_0, y=y_0}$$

Response to a point target $\delta(x-x_1)\delta(y-y_1)\delta(z-z_1)$ is

$$s(x_1-x_0, y_1-y_0) p(2(z'-z_1)/c)$$

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Signal Equation Summary

In general, we can write

$$e_c(t, x_0, y_0) = K \int \int \int R(x, y, z) s(x - x_0, y - y_0) p(t - 2z/c) dx dy dz$$

$$= K \frac{c}{2} \left[R(x, y, ct/2) *** s(-x, -y) p(t) \right] \Big|_{x=x_0, y=y_0}$$

$$e_c(z', x_0, y_0) = K \int \int \int R(x, y, z) s(x - x_0, y - y_0) p(2(z' - z)/c) dx dy dz$$

$$= \left[R(x, y, z') *** s(-x, -y) p(2z'/c) \right] \Big|_{x=x_0, y=y_0}$$

Response to a point target $\delta(x - x_1)\delta(y - y_1)\delta(z - z_1)$ is
 $s(x_1 - x_0, y_1 - y_0) p(2(z' - z_1)/c)$

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Signal Equation Summary

Response to a point target $\delta(x - x_1)\delta(y - y_1)\delta(z - z_1)$ is
 $s(x_1 - x_0, y_1 - y_0) p(2(z' - z_1)/c)$

Thus, $s(x, y)$ determine the lateral response and $p(t)$ determines the depth response.

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Depth Resolution

$p(t) = p(2z/c)$ determines the depth resolution

Pulses are of the form $a(t) \cos(2\pi f_0 t + \theta)$ where $a(t)$ is the envelope function and f_0 is the resonant frequency of the transducer.

The duration of T of $a(t)$ is typically chosen to be about 2 or 3 periods (e.g. $T = 3/f_0$). If the duration is too short, the bandwidth of the pulse will be very large and much of its power will be attenuated.

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c / f_0 = 1.5\lambda$.

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Depth Resolution

The depth resolution is approximately $\Delta z = cT/2 \approx 1.5c / f_0 = 1.5\lambda$.

Example :

For $f_0 = 5$ MHz, $\Delta z = (1.5)(1500 \text{ m/s}) / (5 \times 10^6 \text{ Hz}) = 0.45 \text{ mm}$

Trade - off

Higher $f_0 \Rightarrow$ Smaller $\Delta z \Rightarrow$ but more attenuation

Example : Assume 1dB/cm/MHz

For 10 cm depth, 20 cm roundtrip path length.

At 1 MHz 20 dB of attenuation \Rightarrow Attenuation = 0.1

At 10 MHz 200 dB of attenuation \Rightarrow Attenuation = 1×10^{-10}

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