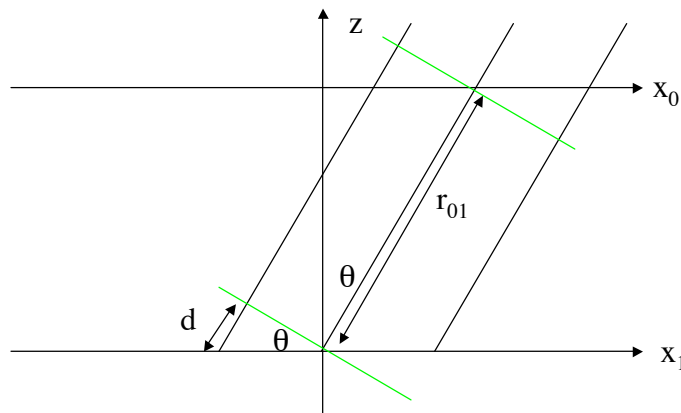


Bioengineering 280A  
Principles of Biomedical Imaging

Fall Quarter 2005  
Ultrasound Lecture 2

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### Plane Wave (Fraunhofer) Approximation



$$d = -x_1 \sin \theta$$
$$\sin \theta = \frac{x_0}{r_{01}} \approx \frac{x_0}{z}$$
$$d \approx -\frac{x_0 x_1}{z}$$

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## Plane Wave Approximation

$$\begin{aligned} \frac{1}{r} \exp(jkr) &\approx \frac{1}{z} \exp(jk(z+d)) = \frac{1}{z} \exp\left(j \frac{2\pi}{\lambda} \left(z - \frac{x_0 x_1}{z}\right)\right) \\ U(x_0) &= \int_{-\infty}^{\infty} s(x_1) \frac{1}{r} \exp(jkr) dx_1 \\ &\approx \int_{-\infty}^{\infty} s(x_1) \frac{1}{z} \exp\left(j \frac{2\pi}{\lambda} \left(z - \frac{x_0 x_1}{z}\right)\right) dx_1 \\ &= \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) \int_{-\infty}^{\infty} s(x_1) \exp\left(-\frac{j 2\pi x_0 x_1}{\lambda z}\right) dx_1 \\ &= \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) \int_{-\infty}^{\infty} s(x_1) \exp(-j 2\pi k_x x_1) dx_1 \\ &= \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) F[s(x)]_{k_x = \frac{x_0}{\lambda z}} \end{aligned}$$

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## Plane Wave Approximation

In general

$$U(x_0, y_0) = \frac{1}{z} \exp\left(j \frac{2\pi z}{\lambda}\right) F[s(x, y)]_{k_x = \frac{x_0}{\lambda z}, k_y = \frac{y_0}{\lambda z}}$$

*Example*

$$s(x, y) = \text{rect}(x/D) \text{rect}(y/D)$$

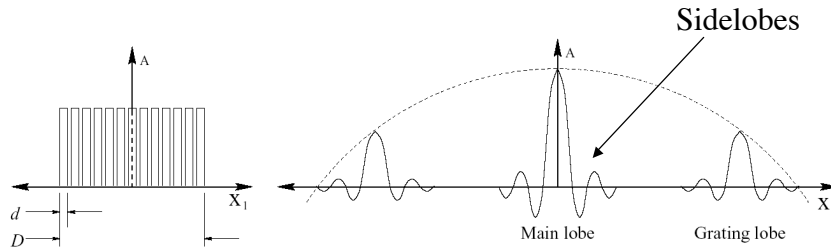
$$\begin{aligned} U(x_0, y_0) &= \frac{1}{z} \exp(jkz) D^2 \text{sinc}(Dk_x) \text{sinc}(Dk_y) \\ &= \frac{1}{z} \exp(jkz) D^2 \text{sinc}\left(D \frac{x_0}{\lambda z}\right) \text{sinc}\left(Dk_y \frac{y_0}{\lambda z}\right) \end{aligned}$$

$$\text{Zeros occur at } x_0 = \frac{n\lambda z}{D} \text{ and } y_0 = \frac{n\lambda z}{D}$$

$$\text{Beamwidth of the sinc function is } \frac{\lambda z}{D}$$

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## Example



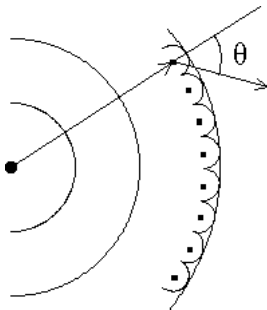
$$\text{rect}\left(\frac{x}{D}\right) \left[ \text{rect}\left(\frac{x}{d}\right) * \frac{1}{d} \text{comb}\left(\frac{x}{d}\right) \right] \Leftrightarrow D \text{sinc}(Dk_x) * [d \text{sinc}(dk_x) \text{comb}(dk_x)]$$

Question: What should we do to reduce the sidelobes?

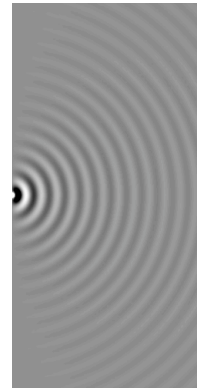
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## Huygen's Principle



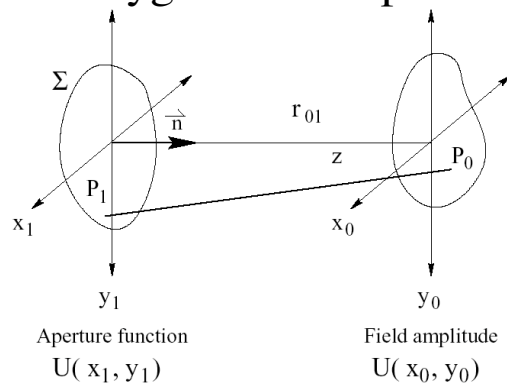
<http://www.fink.com/thesis/chapter2.html>



<http://www.cbem.imperial.ac.uk/ardan/diff/hfw.html>

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## Huygen's Principle



$$U(P_0) = \iint_{\Sigma} h(P_0, P_1) U(P_1) ds$$

Wavenumber  $k = \frac{2\pi}{\lambda}$

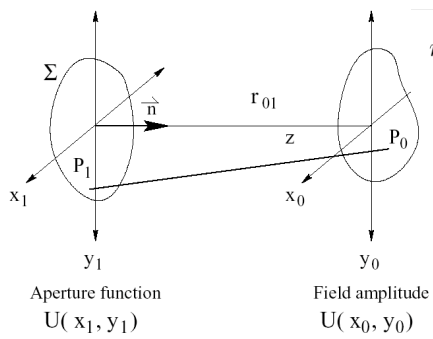
where  $h(P_0, P_1) = \frac{1}{j\lambda} \frac{\exp(jkr_{01})}{r_{01}} \cos(\vec{n}, \vec{r}_{01})$

Oliquity Factor

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## Small-Angle (paraxial) Approximation



$$r_{01} = \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$\cos(\vec{n}, \vec{r}_{01}) \approx 1$$

$$r_{01} \approx z$$

$$h(x_0, y_0; x_1, y_1) \approx \frac{1}{j\lambda z} \exp(jkr_{01})$$

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## Fresnel Approximation

$$\begin{aligned}r_{01} &= \sqrt{z^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= z \sqrt{1 + \left(\frac{(x_1 - x_0)^2}{z^2}\right) + \left(\frac{(y_1 - y_0)^2}{z^2}\right)} \\ &\approx z \left[ 1 + \frac{1}{2} \left(\frac{(x_1 - x_0)^2}{z^2}\right) + \frac{1}{2} \left(\frac{(y_1 - y_0)^2}{z^2}\right) \right]\end{aligned}$$

Approximates spherical wavefront with a parabolic phase profile

$$h(x_0, y_0; x_1, y_1) \approx \frac{\exp(jkz)}{j\lambda z} \exp \left[ \frac{jk}{2z} \left[ (x_1 - x_0)^2 + (y_1 - y_0)^2 \right] \right]$$

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## Fresnel Approximation

$$\begin{aligned}U(x_0, y_0) &= \iint \frac{\exp(jkz)}{j\lambda z} \exp \left( \frac{jk}{2z} \left[ (x_1 - x_0)^2 + (y_1 - y_0)^2 \right] \right) s(x_1, y_1) dx_1 dy_1 \\ &= \frac{\exp(jkz)}{j\lambda z} \left( s(x_0, y_0) ** \exp \left( \frac{jk}{2z} (x_0^2 + y_0^2) \right) \right)\end{aligned}$$

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## Fraunhofer Approximation

$$\begin{aligned}
 kr_{01} &\approx kz \left( 1 + \frac{1}{2} \left( \frac{x_1 - x_0}{z} \right)^2 + \frac{1}{2} \left( \frac{y_1 - y_0}{z} \right)^2 \right) \\
 &= kz \left( 1 + \frac{1}{2z^2} (x_1^2 - 2x_1x_0 + x_0^2) + \frac{1}{2z^2} (y_1^2 - 2y_1y_0 + y_0^2) \right) \\
 &= kz + \frac{k}{2z} (x_1^2 + y_1^2) + \frac{k}{2z} (x_0^2 + y_0^2) - \frac{k}{z} (x_1x_0 + y_1y_0) \\
 &\approx kz + \frac{k}{2z} (x_0^2 + y_0^2) - \frac{k}{z} (x_1x_0 + y_1y_0)
 \end{aligned}$$

Assume this term is negligible.

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## Fraunhofer Condition

Phase term due to position on transducer is  $\frac{k}{2z} (x_1^2 + y_1^2)$

Far-field condition is

$$\begin{aligned}
 \frac{k}{2z} (x_1^2 + y_1^2) &\ll 1 \\
 z \gg \frac{k}{2} (x_1^2 + y_1^2) &= \frac{\pi}{\lambda} (x_1^2 + y_1^2)
 \end{aligned}$$

For a square  $D \times D$  transducer,  $x_1^2 + y_1^2 = D^2 / 4$

$$z \gg \frac{\pi D^2}{4\lambda} \approx \frac{D^2}{\lambda}$$

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# Fraunhofer Approximation

$$U(x_0, y_0) \approx \frac{\exp(jkz) \exp\left[\frac{jk}{2z}(x_0^2 + y_0^2)\right]}{j\lambda z} \iint_{-\infty}^{\infty} U(x_1, y_1) \exp\left[-\frac{j2\pi}{\lambda z}(x_0 x_1 + y_0 y_1)\right] dx_1 dy_1$$

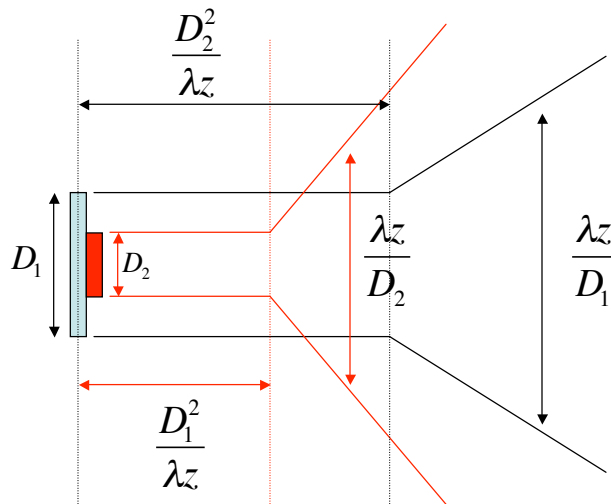
Quadratic phase term

Fourier transform of the source with

$$k_x = \frac{x_0}{\lambda z} \quad k_y = \frac{y_0}{\lambda z}$$

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## Transducer Dimension

Goal : Operate in the Fresnel Zone

$$z < D^2 / \lambda$$

$$D_{opt} \approx \sqrt{\lambda z_{max}}$$

*Example*

$$z_{max} = 20 \text{ cm}$$

$$\lambda = 0.5 \text{ mm}$$

$$D_{opt} = 1 \text{ cm}$$

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## Focusing in Fresnel Zone

$$\begin{aligned} U(x_0, y_0) &= \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}((x_1 - x_0)^2 + (y_1 - y_0)^2)\right) s(x_1, y_1) dx_1 dy_1 \\ &= \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}((x_1^2 + y_1^2) + (x_0^2 + y_0^2) - 2(x_1 x_0 + y_1 y_0))\right) s(x_1, y_1) dx_1 dy_1 \\ &= \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}(x_0^2 + y_0^2)\right) \iint \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}(x_1^2 + y_1^2)\right) \exp\left(-\frac{jk}{z}(x_1 x_0 + y_1 y_0)\right) s(x_1, y_1) dx_1 dy_1 \end{aligned}$$



Use time delays to compensate for this phase term

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}(x_0^2 + y_0^2)\right) F\left[\exp\left(\frac{jk}{2z}(x_1^2 + y_1^2)\right) s(x_1, y_1)\right]$$

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## Focusing in Fresnel Zone

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z} \exp\left(\frac{jk}{2z}(x_0^2 + y_0^2)\right) F\left[\exp\left(\frac{jk}{2z}(x_1^2 + y_1^2)\right) s(x_1, y_1)\right]$$

Make  $s(x_1, y_1) = s_0(x_1, y_1) \exp\left(-\frac{jk}{2z_0}(x_1^2 + y_1^2)\right)$

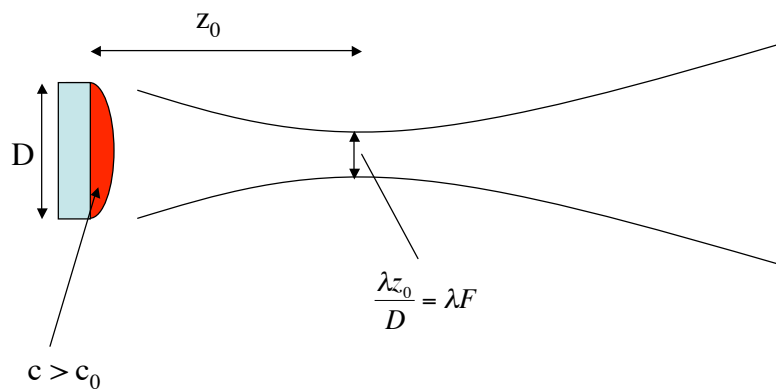
At the focal depth  $z = z_0$

$$U(x_0, y_0) = \frac{\exp(jkz_0)}{j\lambda z_0} \exp\left(\frac{jk}{2z_0}(x_0^2 + y_0^2)\right) F[s(x_1, y_1)]$$

Beamwidth at the focal depth is:  $\frac{\lambda z_0}{D}$

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## Acoustic Lens



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## Depth of Focus

When  $z \neq z_0$ , the phase term is  $\Delta\Phi = \exp\left(-\frac{jk}{2z_0}(x_1^2 + y_1^2)\right)\exp\left(-\frac{jk}{2z}(x_1^2 + y_1^2)\right)$   
and the lens is not perfectly focused.

Consider variation in the  $x$  - direction.

$$\Delta\Phi = \frac{kx^2}{2} \left( \frac{1}{z} - \frac{1}{z_0} \right)$$

For transducer of size  $D$ ,  $\frac{x^2}{2} = \frac{D^2}{4}$

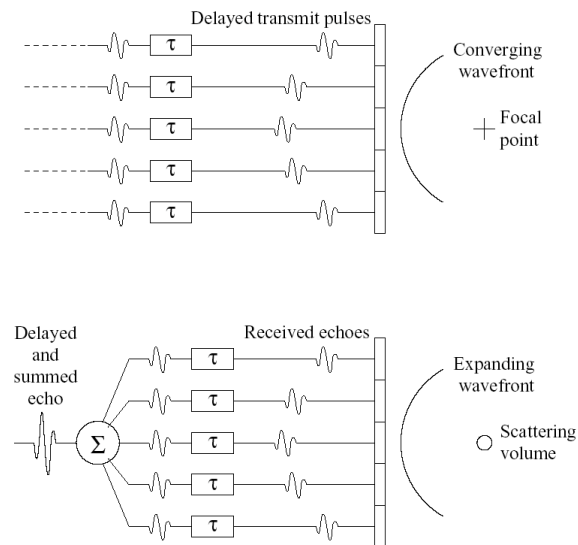
If we want  $|\Delta\Phi| = \left| \frac{\pi D^2}{2\lambda} \left( \frac{1}{z} - \frac{1}{z_0} \right) \right| < 1$  radian then

$$\left| \frac{1}{z} - \frac{1}{z_0} \right| < \frac{2\lambda}{\pi D^2}$$

The larger the  $D$ , the smaller the depth of focus.

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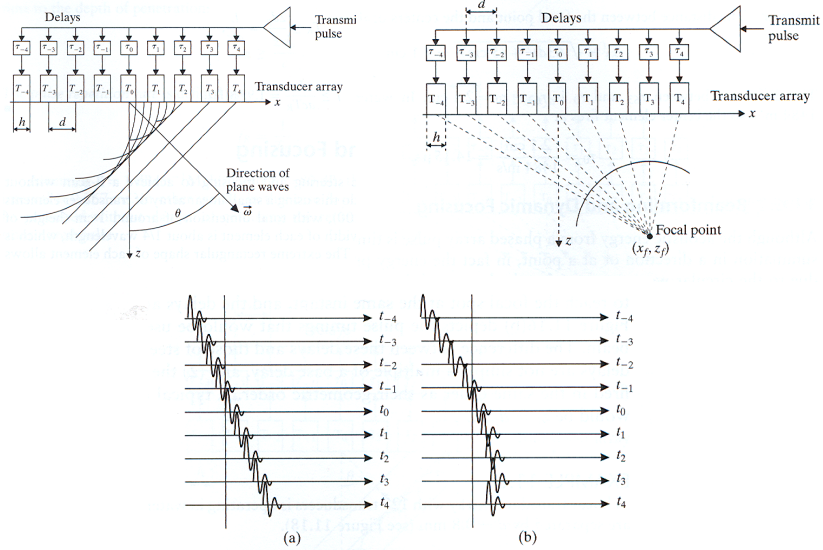
## Focusing with Phased Array



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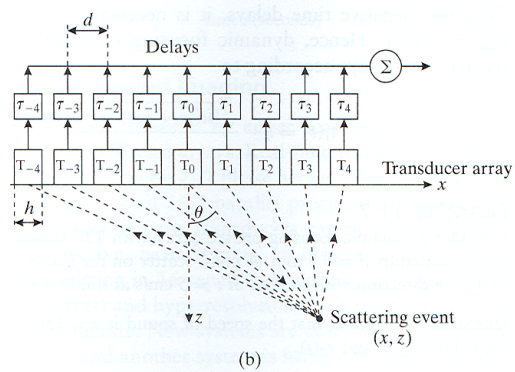
# Focusing and Steering



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# Dynamic Focusing



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