Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
X-Rays/CT Lecture 1

Topics

- X-Rays
- Computed Tomography
- Direct Inverse and Iterative Inverse
- Backprojection
- Projection Theorem
- Filtered Backprojection

EM spectrum

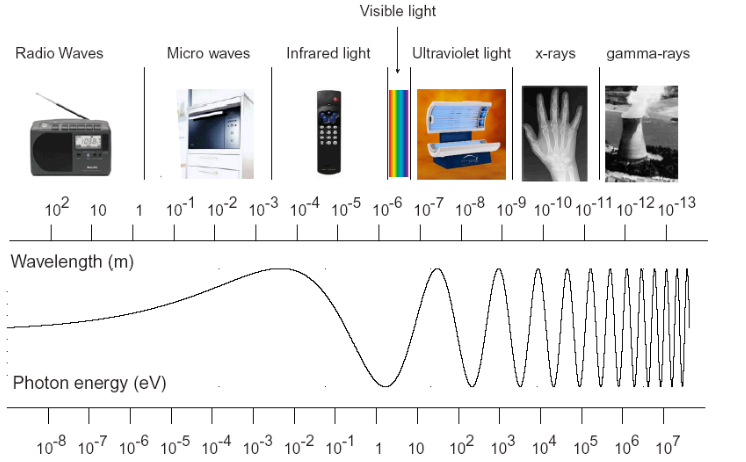

Figure 4.1: *The electromagnetic spectrum.*

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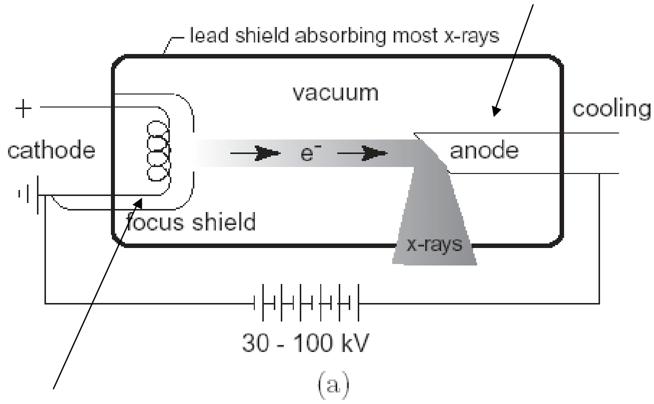
X-Ray Tube

Usually tungsten is used for anode
Molybdenum for mammography



Tungsten filament heated to about 2200 C leading to thermionic emission of electrons.

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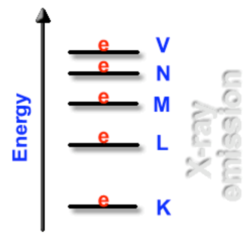
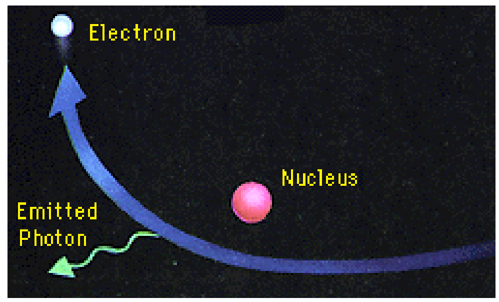
X-Ray Production



Characteristic Radiation

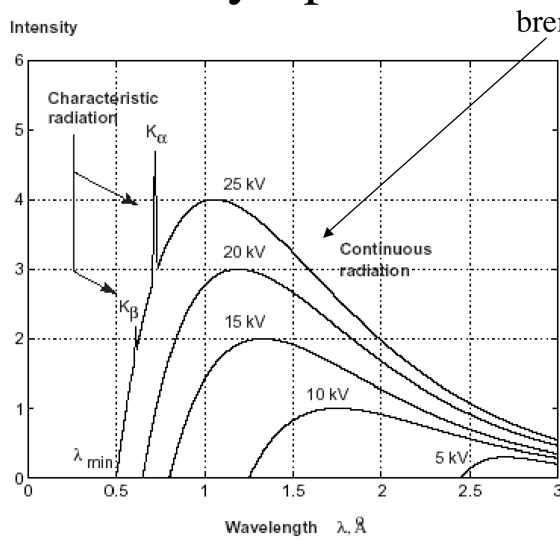
Bremsstrahlung
(braking radiation)



http://www.scienceofspectroscopy.info/theory/ADVANCED/x_ray.htm

X-Ray Spectrum

bremsstrahlung



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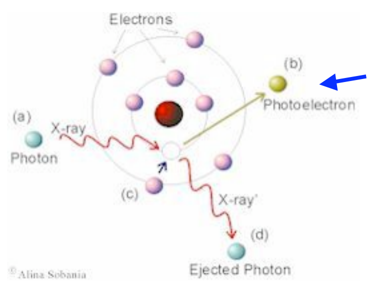
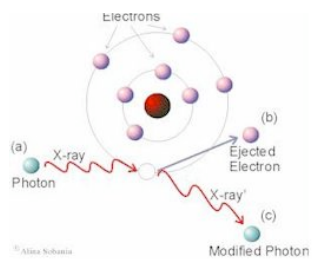
Interaction with Matter

Typical energy range for diagnostic x-rays is below 200 keV. The two most important types of interaction are photoeletric absorption and Compton scattering.



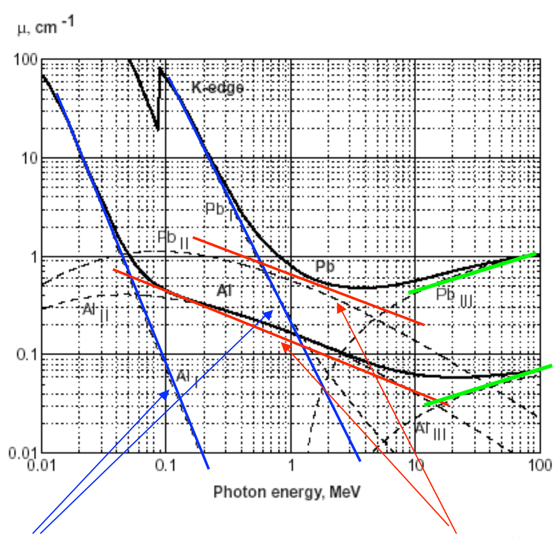
Photoelectric effect dominates at low x-ray energies and high atomic numbers.

Compton scattering dominates at high x-ray energies and low atomic numbers, not much contrast



http://www.eee.ntu.ac.uk/research/vision/asobania

Interaction with Matter

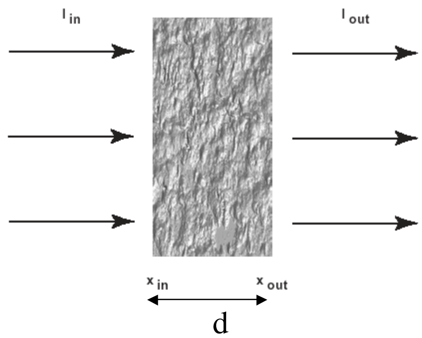


Pair Production

Photoelectric absorption

Compton Scattering

Attenuation



For single-energy x-rays passing through a homogenous object:

$$I_{out} = I_{in} \exp(-\mu d)$$

Linear attenuation coefficient

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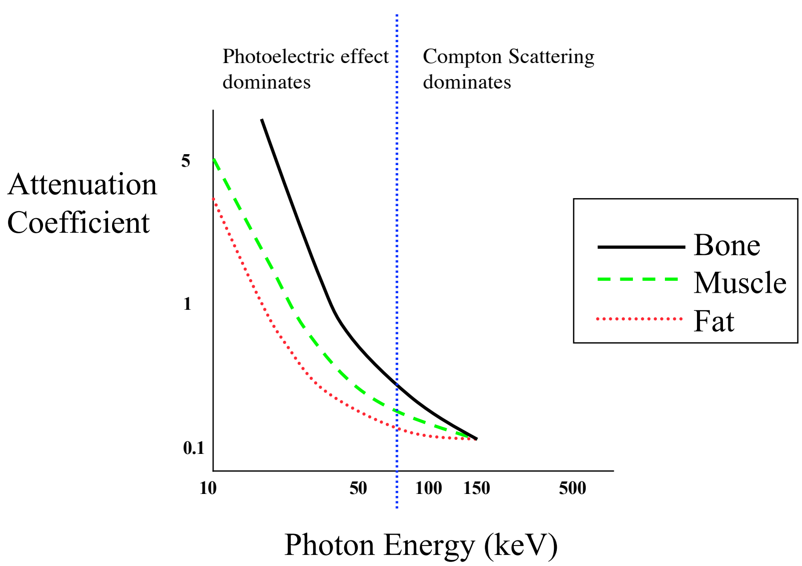
Attenuation



Adapted from www.cis.rit.edu/class/simg215/xrays.ppt

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Half Value Layer

X-ray energy (keV)	HVL, muscle (cm)	HVL Bone (cm)
30	1.8	0.4
50	3.0	1.2
100	3.9	2.3
150	4.5	2.8

In chest radiography, about 90% of x-rays are absorbed by body. Average energy from a tungsten source is 68 keV. However, many lower energy beams are absorbed by tissue, so average energy is higher. This is referred to as beam-hardening, and reduces the contrast.

Values from Webb 2003

Attenuation

For an inhomogenous object:

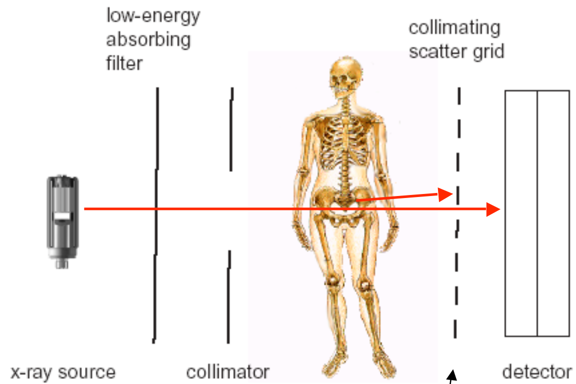
$$I_{out} = I_{in} \exp\left(-\int_{x_{in}}^{x_{out}} \mu(x)dx\right)$$

Integrating over energies

$$I_{out} = \int_0^\infty \sigma(E) \exp\left(-\int_{x_{in}}^{x_{out}} \mu(E,x)dx\right)dE$$

Intensity distribution of beam

X-Ray Imaging Chain



Reduces effects of Compton scattering

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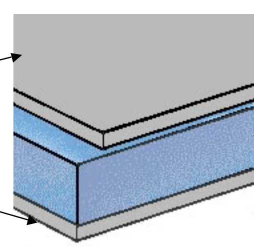
X-ray film

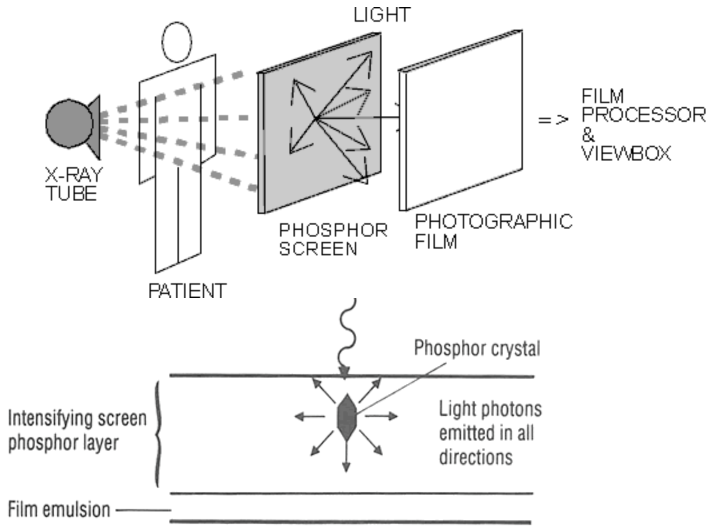
Emulsion with
silver halide crystals
Each layer
~ 10 μm



Flexible base
~ 150 μm

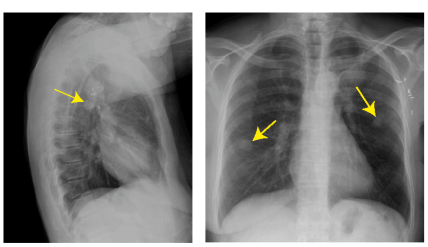
Silver halide crystals absorb optical energy. After development, crystals that have absorbed enough energy are converted to metallic silver and look dark on the screen. Thus, parts in the object that attenuate the x-rays will look brighter.

Intensifying Screen

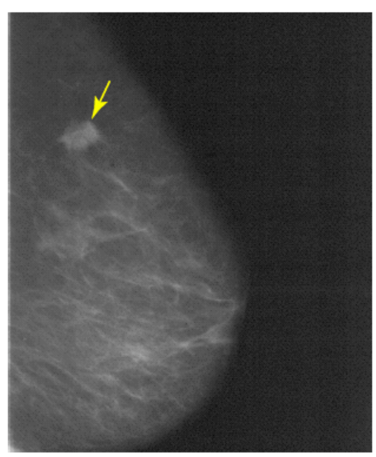


http://learntech.uwe.ac.uk/radiography/RScience/imaging_principles_d/diagimage11.htm
http://www.sunnybrook.utoronto.ca:8080/~selenium/xray.html#Film

X-Ray Examples



(a) (b)



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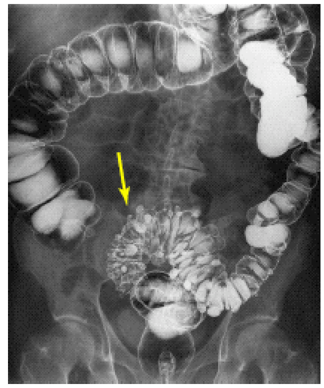
X-Ray w/ Contrast Agents



Angiogram using an iodine-based contrast agent.
K-edge of iodine is 33.2 keV

Barium Sulfate
K-edge of Barium is 37.4 keV

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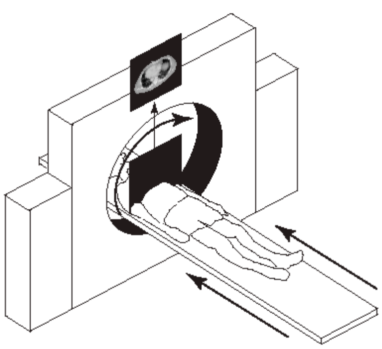
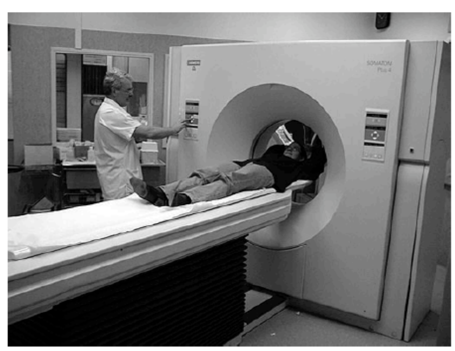
Computed Tomography





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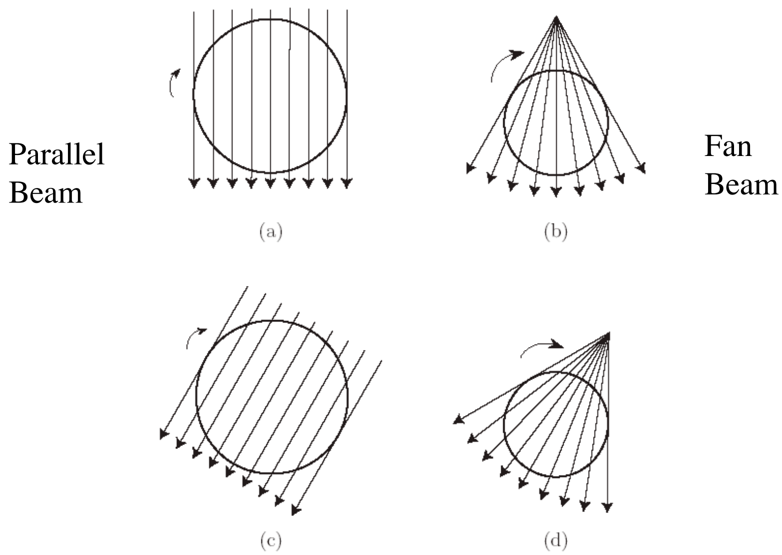
Computed Tomography



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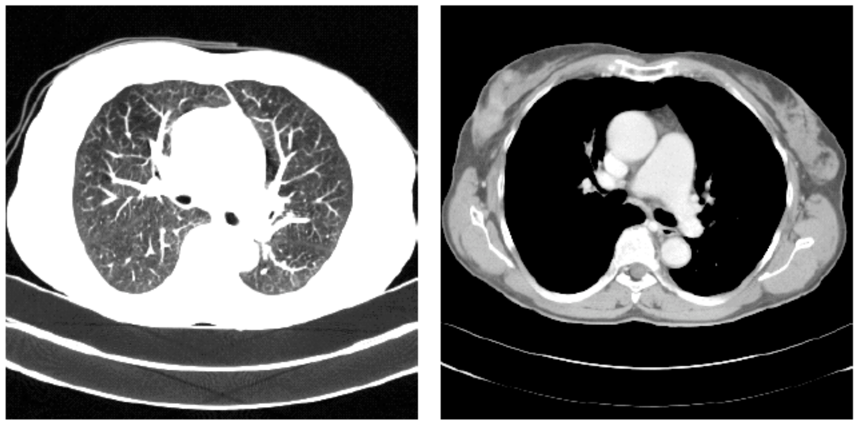
CT Number

$$\text{CT\_number} = \frac{\mu - \mu_{water}}{\mu_{water}} \times 1000$$

Measured in Hounsfield Units (HU)

Air: -1000 HU
Soft Tissue: -100 to 60 HU
Cortical Bones: 250 to 1000 HU

CT Display



(a) (b)

Figure 5.4: *CT-image of the chest with different window/level settings:(a) for the lungs (window 1500 and level -500) and (b) for the soft tissues (window 350 and level 50).*

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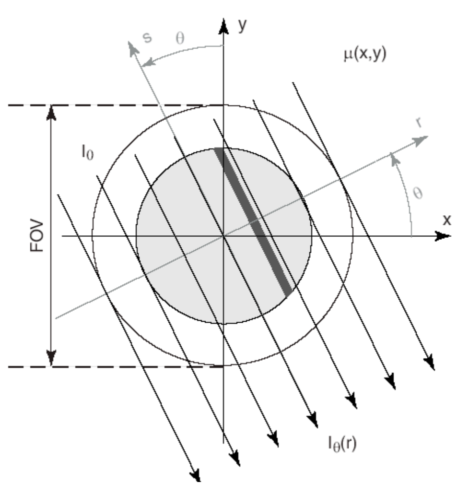
Projections



$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$$

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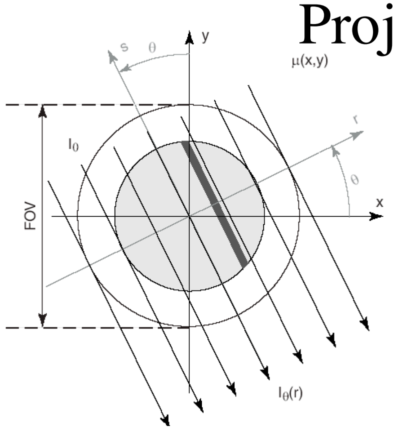
Projections



$$I_\theta(r) = I_0 \exp\left(-\int_{L_{r,\theta}} \mu(x,y)\,ds\right)$$

$$= I_0 \exp\left(-\int_{L_{r,\theta}} \mu(r\cos\theta - s\sin\theta, r\sin\theta + s\cos\theta)\,ds\right)$$

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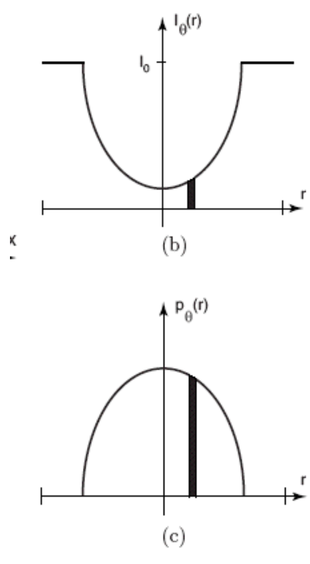
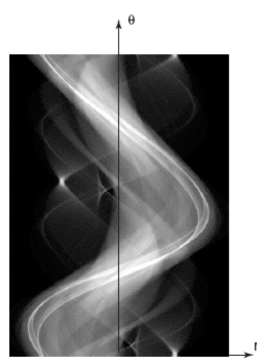
Projections



$$I_\theta(r) = I_0 \exp\left(-\int_{L_{r,\theta}} \mu(r\cos\theta - s\sin\theta, r\sin\theta + s\cos\theta)\,ds\right)$$

$$p_\theta(r) = -\ln\frac{I_\theta(r)}{I_0}$$

$$= \int_{L_{r,\theta}} \mu(r\cos\theta - s\sin\theta, r\sin\theta + s\cos\theta)\,ds$$



Sinogram

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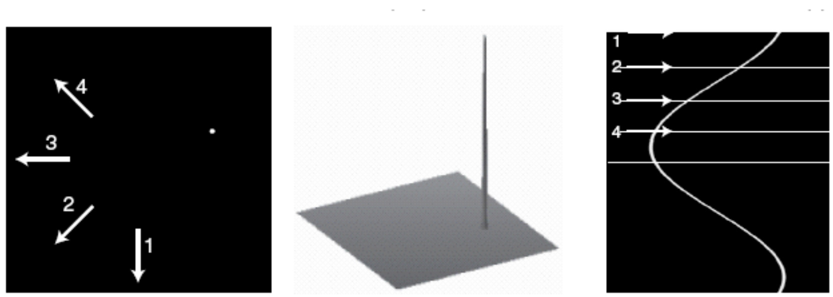
Sinogram



(a) (b) (c)

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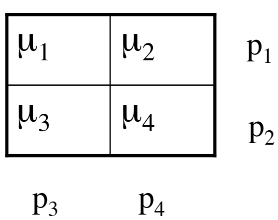
Direct Inverse Approach

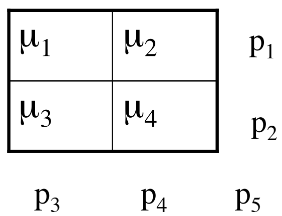


$p_1 = \mu_1 + \mu_2$
$p_2 = \mu_3 + \mu_4$
$p_3 = \mu_1 + \mu_3$
$p_4 = \mu_2 + \mu_4$

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix}$$

4 equations, 4 unknowns.
Are these the correct equations to use?

Direct Inverse Approach

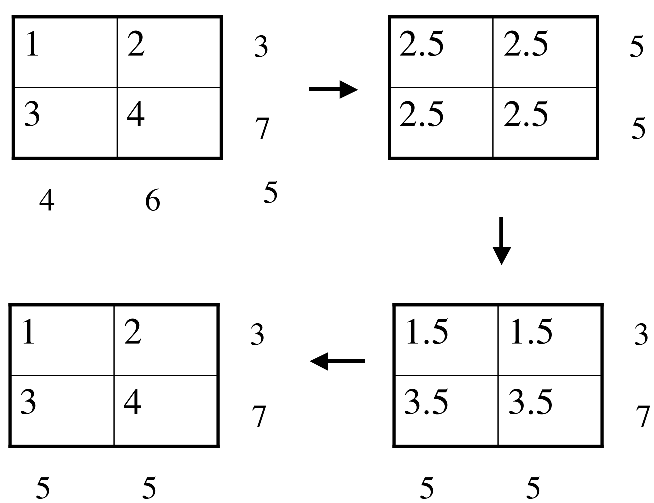


$p_1 = \mu_1 + \mu_2$
$p_2 = \mu_3 + \mu_4$
$p_3 = \mu_1 + \mu_3$
$p_5 = \mu_1 + \mu_4$

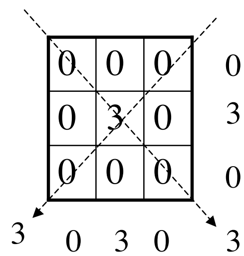
$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix}$$

4 equations, 4 unknowns. These are linearly independent now.
In general for a NxN image, N^2 unknowns, N^2 equations.
This requires the inversion of a N^2xN^2 matrix
For a high-resolution 512x512 image, N^2=262144 equations.
Requires inversion of a 262144x262144 matrix!
Inversion process sensitive to measurement errors.

Iterative Inverse Approach
Algebraic Reconstruction Technique (ART)



Backprojection



$$\begin{bmatrix}0&0&0\\1&1&1\\0&0&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&0\\1&2&1\\0&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&1&0\\1&3&1\\0&1&1\end{bmatrix}\rightarrow\begin{bmatrix}1&1&1\\1&4&1\\1&1&1\end{bmatrix}$$

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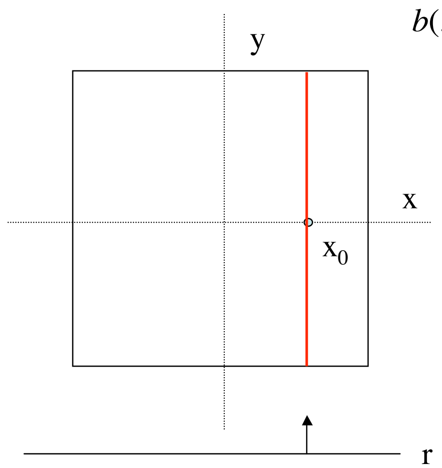
Backprojection



$$b(x,y) = B\{p(r,\theta)\}$$
$$= \int_0^{\pi} p(x\cos\theta + y\sin\theta, \theta)\, d\theta$$

$$b(x_0,y) = p(r,\theta=0)\Delta\theta$$
$$= p(x_0)\Delta\theta$$

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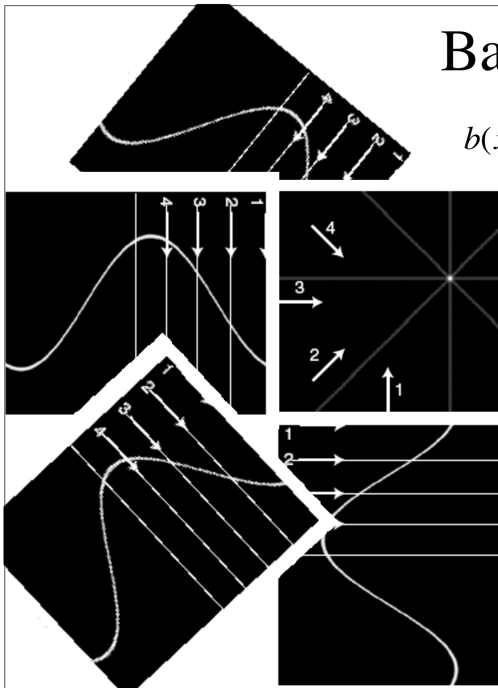

Backprojection

$$b(x,y) = B\{p(r,\theta)\}$$
$$= \int_0^{\pi} p(x\cos\theta + y\sin\theta, \theta)\, d\theta$$

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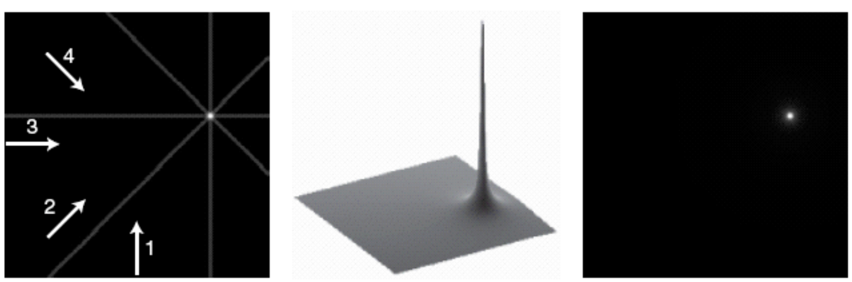
Backprojection



$$b(x,y) = B\{p(r,\theta)\} = \int_0^{\pi} p(x\cos\theta + y\sin\theta, \theta)\, d\theta$$

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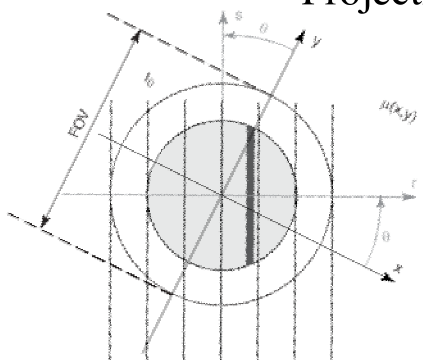
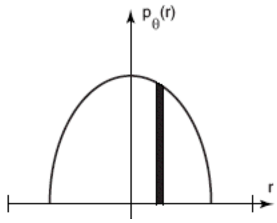
Projection Theorem



$$
\begin{aligned}
U(k_x,0) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x,y)e^{-j2\pi(k_x x+k_y y)}dxdy \\
&= \int_{-\infty}^{-\infty}\left[\int_{-\infty}^{-\infty}\mu(x,y)dy\right]e^{-j2\pi k_x x}dx \\
&= \int_{-\infty}^{-\infty}p_0(x)e^{-j2\pi k_x x}dx \\
&= \int_{-\infty}^{-\infty}p_0(r)e^{-j2\pi kr}dr
\end{aligned}
$$



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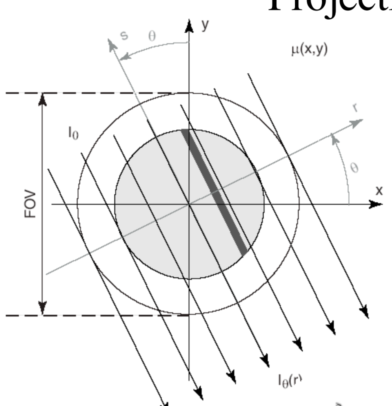
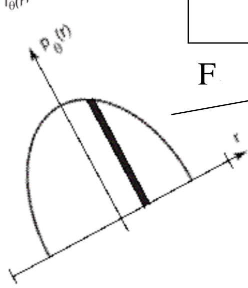
Projection Theorem



$$
\begin{aligned}
U(k_x,k_y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x,y)e^{-j2\pi(k_x x+k_y y)}dxdy \\
&= F_{2D}\left[\mu(x,y)\right]
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
&U(k_x,k_y) = P(k,\theta) \\
&k_x = k\cos\theta \\
&k_y = k\sin\theta \\
&k = \sqrt{k_x^2 + k_y^2}
\end{aligned}
}
$$



F

$$P(k,\theta) = \int_{-\infty}^{\infty}p_\theta(r)e^{-j2\pi kr}dr$$

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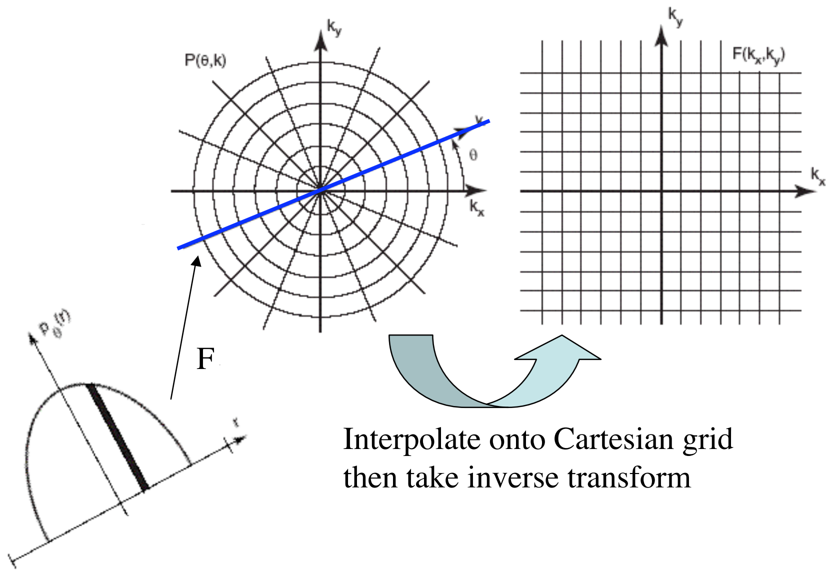
Fourier Reconstruction



Interpolate onto Cartesian grid
then take inverse transform

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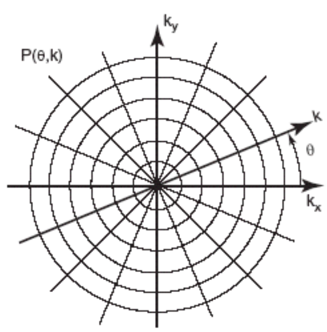
Polar Version of Inverse FT



$$
\begin{aligned}
\mu(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(k_x,k_y)e^{j2\pi(k_x x + k_y y)}dk_x dk_y \\
&= \int_0^{2\pi}\int_0^{\infty} U(k,\theta)e^{j2\pi(k\cos\theta x + k\sin\theta y)}k\,dk\,d\theta \\
&= \int_0^{\pi}\int_{-\infty}^{\infty} U(k,\theta)e^{j2\pi(xk\cos\theta + yk\sin\theta)}|k|\,dk\,d\theta
\end{aligned}
$$

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Filtered Backprojection

$$\mu(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} U(k,\theta)e^{j2\pi(xk\cos\theta \; + yk\sin\theta)}|k|\,dk\,d\theta$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} |k|U(k,\theta)e^{j2\pi kr}\,dk\,d\theta$$

$$= \int_0^{\pi} u^*(r,\theta)\,d\theta$$

Backproject a filtered projection

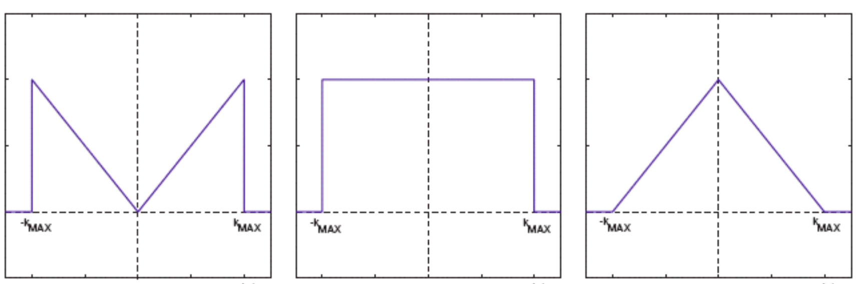
$$where\ r = x\cos\theta + y\sin\theta$$

$$u^*(r,\theta) = \int_{-\infty}^{\infty}|k|U(k,\theta)e^{j2\pi kr}\,dk$$

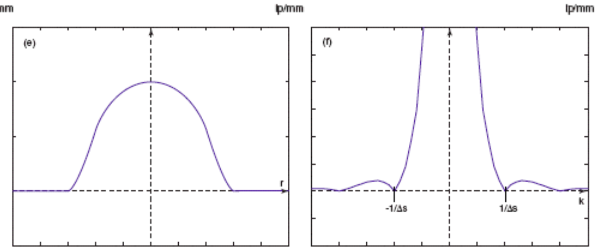
$$= u(r,\theta) * F^{-1}\big[|k|\big]$$

$$= u(r,\theta) * q(r)$$

Ram-Lak Filter

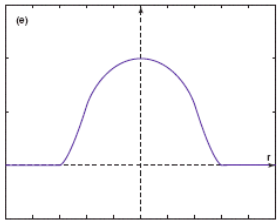


k_{max}=1/Δs

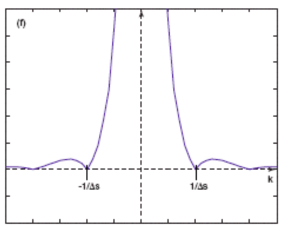


Reconstruction Path

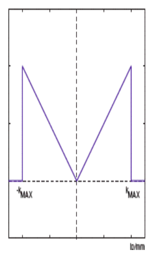
Projection



F ↓



x



$\xrightarrow{F^{-1}}$ Filtered Projection → Back-Project

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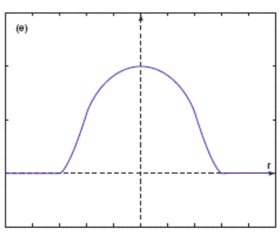
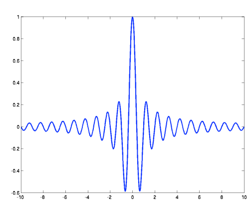
Reconstruction Path

Projection



*



→ Filtered Projection

↓

Back-Project

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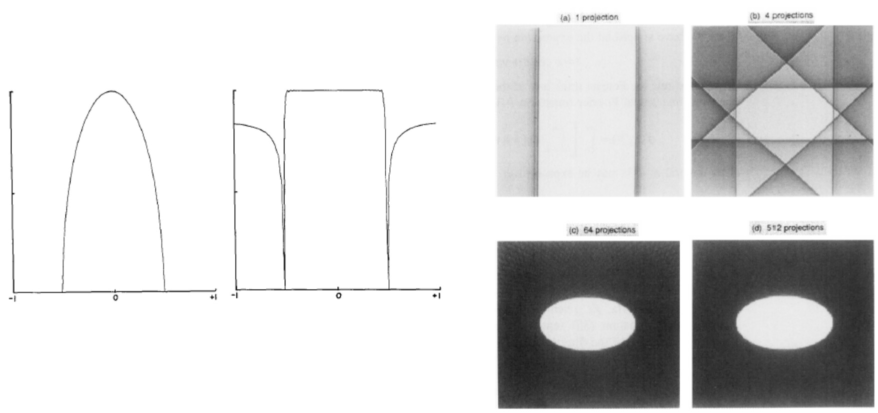
Example



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Kak and Slaney

Example

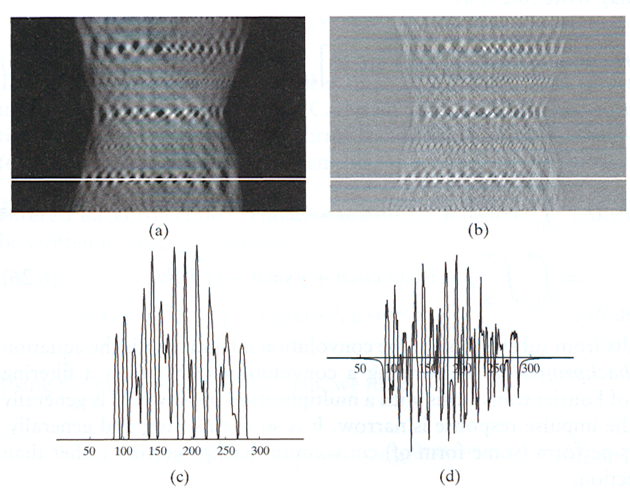

Figure 6.15
Convolution step:
(a) Original sinogram;
(b) filtered sinogram;
(c) profile of sinogram row [white line in (a)]; and
(d) profile of filtered sinogram row [white line in (b)].

TT Liu, BE280A, UCSD Fall 2005

Prince and Links 2005

Example

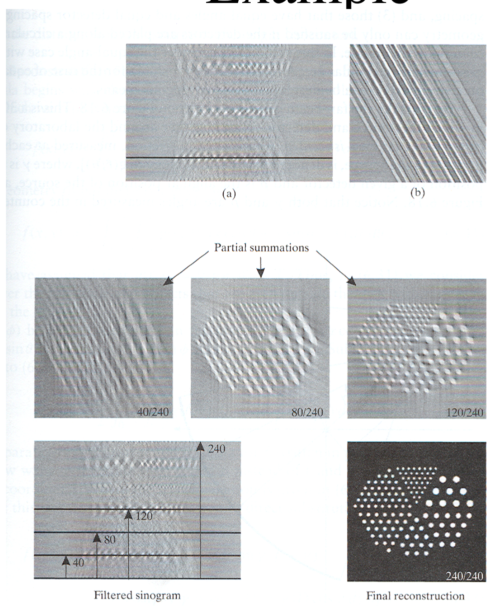

Figure 6.16
Backprojection step.

Figure 6.17
Summation step.

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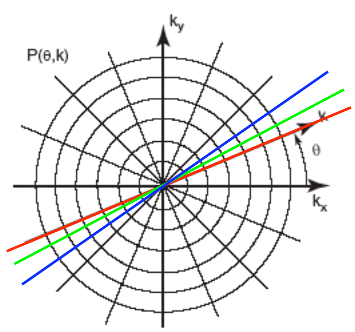
Fourier Interpretation



$$\text{Density} \approx \frac{N}{\text{circumference}} \approx \frac{N}{2\pi|k|}$$

Low frequencies are oversampled. So to compensate for this, multiply the k-space data by |k| before inverse transforming.

Kak and Slaney; Suetens 2002

TT Liu, BE280A, UCSD Fall 2005

Additional Filtering

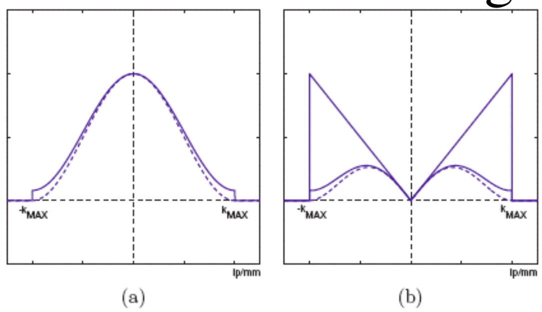
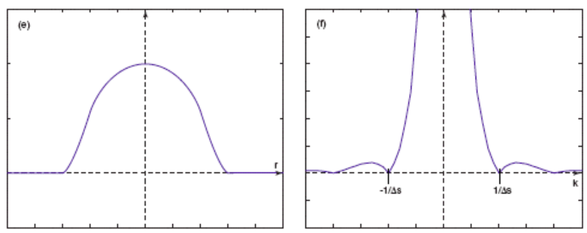


Figure 5.12: *(a) Hamming window with* $\alpha = 0.54$ *and Hanning window (dashed) with* $\alpha = 0.5$. *(b) Ramp filter and its products with a Hamming window and a Hanning window (dashed).*

k_{max}=1/Δs



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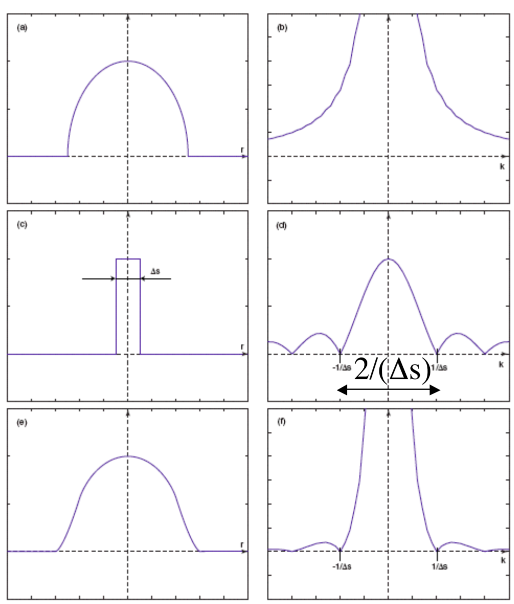
Suetens 2002

Sampling Requirements

Projection

Beam
Width

Smoothed
Projection



W= 2/(Δs)
δ=1/W= Δs/2

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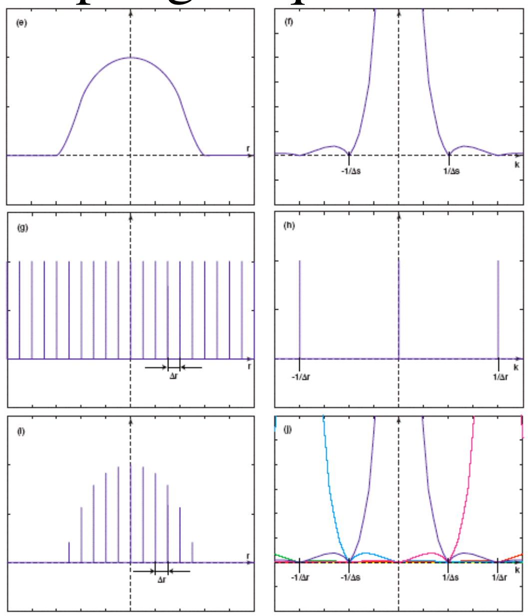
Sampling Requirements

Smoothed Projection

Detectors
$\Delta r \leq \Delta s/2$

Sampled Smooth Projection



Suetens 2002

Sampling Requirements

Size of detector $\Delta r = \delta = 1/W = \Delta s/2$
Number of Detectors $N = FOV/\Delta r$ where $\Delta r \leq \Delta s/2$

Angular Sampling -- how many views?

Want Circumference/(views in 360 degrees) = Δr

$\pi FOV/(views) = \Delta r = FOV/N$

Number of views in 360 degrees = πN

Suetens 2002