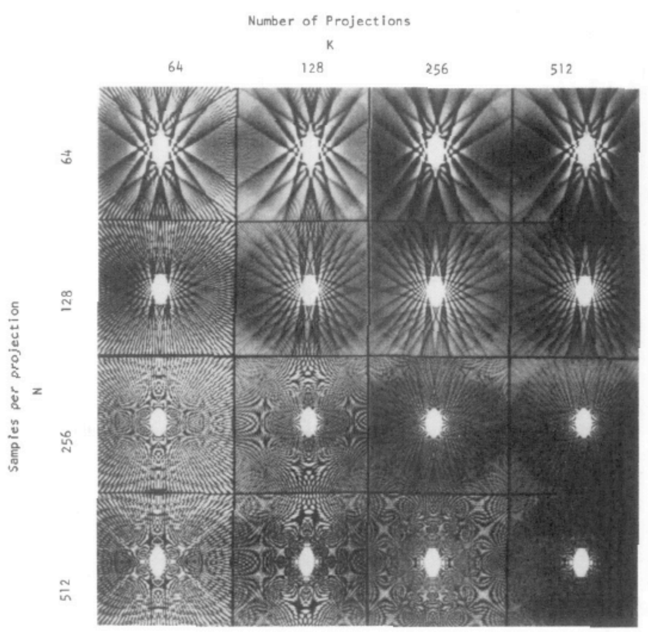
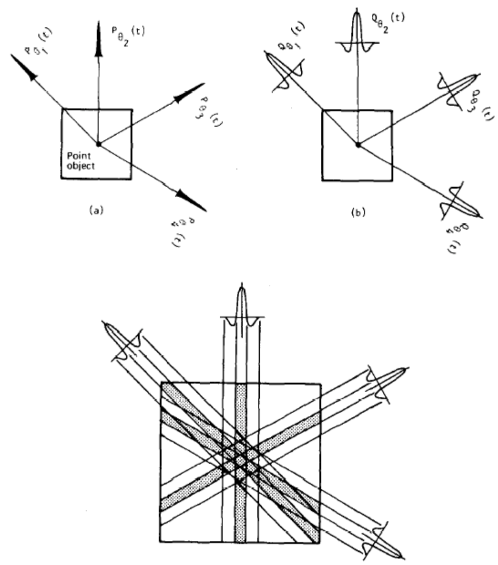
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
X-Rays/CT Lecture 2



Kak and Slaney

View Aliasing



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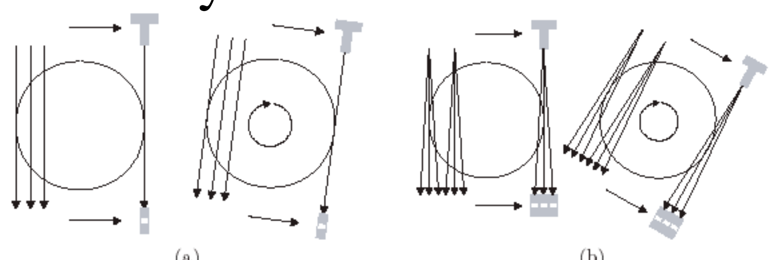
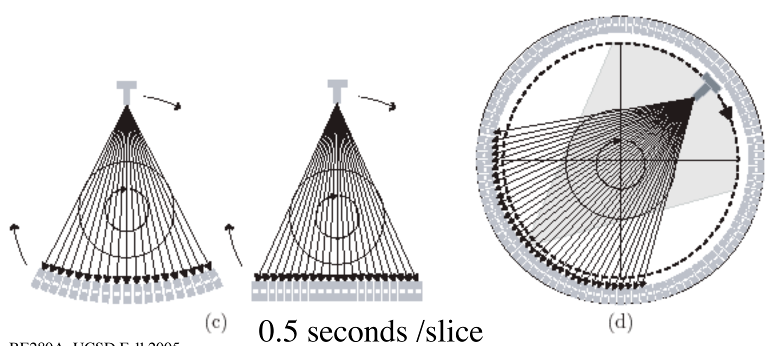
Kak and Slaney

CT System Generations



5 minutes/slice

20 seconds /slice



0.5 seconds /slice

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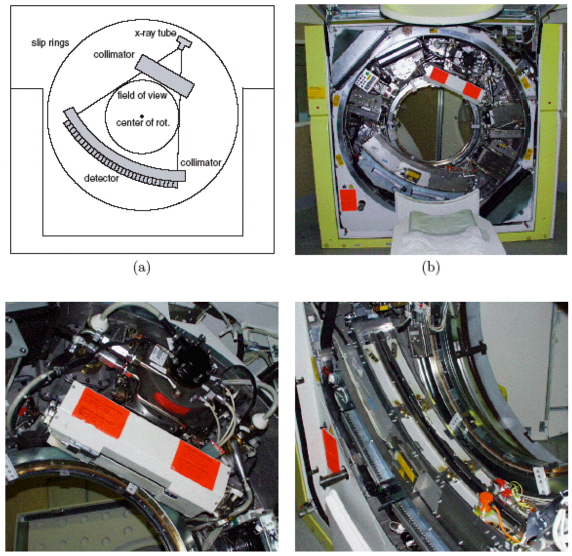
CT System



(a) (b) (c) (d)

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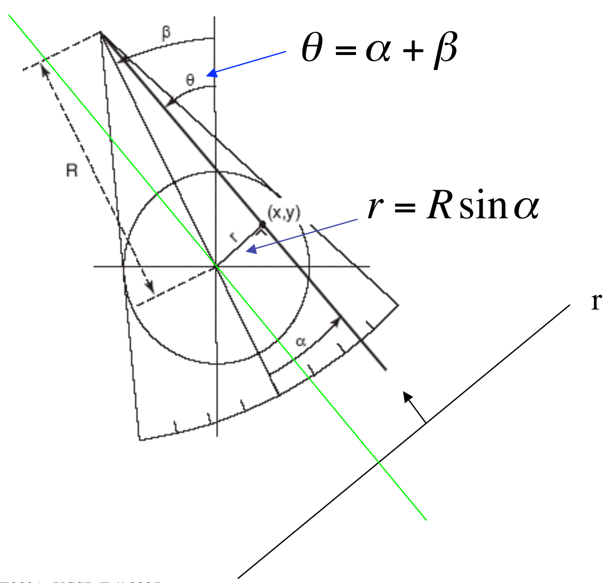
Fan Beam



$$\theta = \alpha + \beta$$

$$r = R \sin \alpha$$

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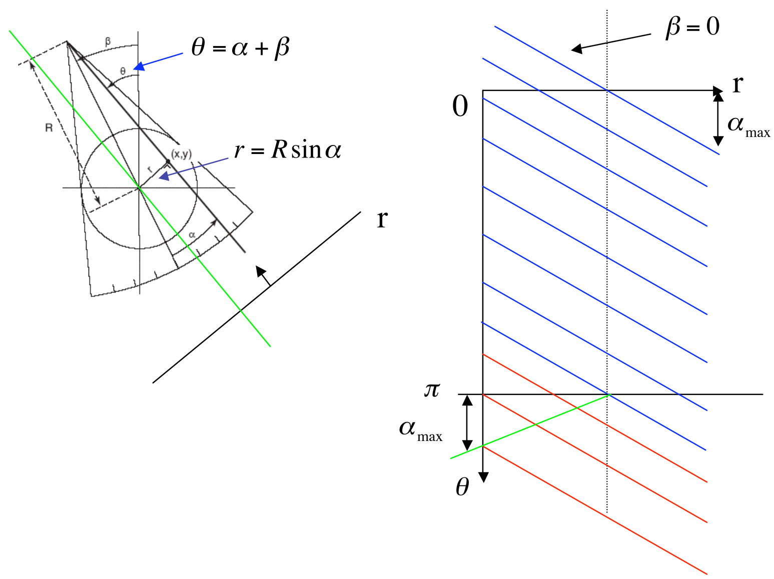
Fan Beam



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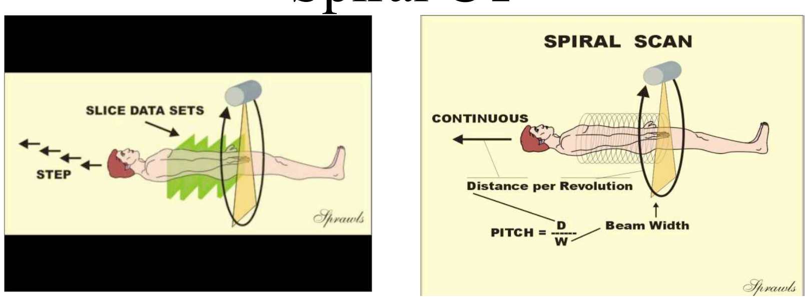
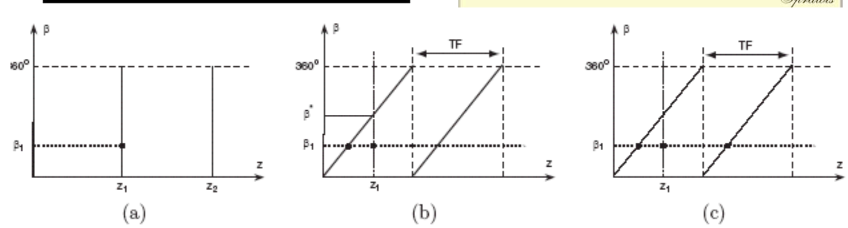
Spiral CT





Nearest Neighbor Interpolation

Linear Interpolation

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From http://www.sprawls.org/resources/CTIMG/classroom.htm
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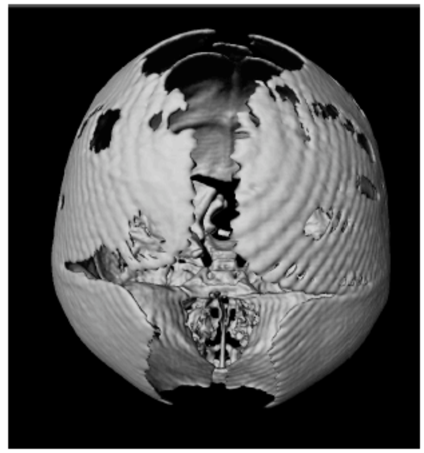
Longitudinal Aliasing in Spiral CT



From http://www.sprawls.org/resources/CTIMG/classroom.htm
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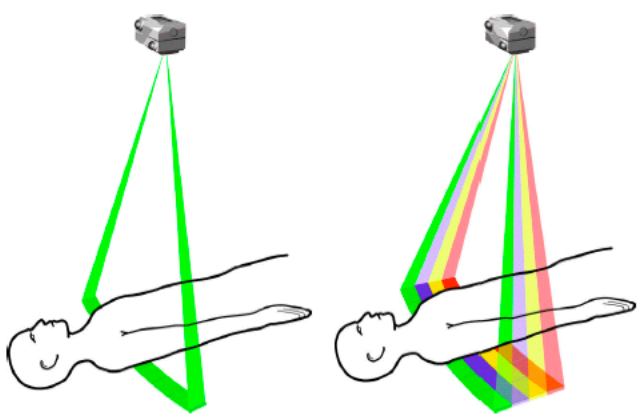
Multislice CT



(a) (b)

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Poisson Process

Events occur at random instants of time at an average rate of λ events per second.

Examples: arrival of customers to an ATM, emission of photons from an x-ray source, lightning strikes in a thunderstorm.

Assumptions:

1) Probability of more than 1 event in an small time interval is small.
2) Probability of event occurring in a given small time interval is independent of another event occuring in other small time intervals.

Poisson Process

$$P\left[N(t)=k\right]=\frac{(\lambda t)^k}{k!}\exp(-\lambda t)$$

λ = Average rate of events per second
λt = Average number of events at time t
λt = Variance in number of events

Probability of interarrival times
$P\left[T>t\right]=e^{-\lambda t}$

Example

A service center receives an average of 15 inquiries per minute. Find the probability that 3 inquiries arrive in the first 10 seconds.

$$\lambda = 15/60 = 0.25$$
$$\lambda t = 0.25(10) = 2.5$$

$$P[N(t=10)=3) = \frac{(2.5)^3}{3!}\exp(-2.5) = .2138$$

Quantum Noise

Fluctuation in the number of photons emitted by the x-ray source and recorded by the detector.

$$P_k = \frac{N_0^k \exp(-N_0)}{k!}$$

P_k : Probability of emitting k photons in a given time interval.

N_0 : Average number of photons emitted in that time interval = λt

Transmitted Photons

$$Q_k = \frac{(pN_0)^k \exp(-pN_0)}{k!}$$

Q_k : Probability of k photons making it through object

N_0 : Average number of photons emitted in that time interval = λt

$p = \exp(-\int \mu dz) =$ probability of proton being transmitted

Example

Over the diagnostic energy range, the photon density is approximately 2.5×10^{10} photons/cm^2 / R where R stands for roentgen (unit for X-ray exposure).

A typical chest x-ray has an exposure of 50 mR.
For transmission in regions devoid of bone,
$p = \exp(-\int \mu dz) \approx 0.05$
What are the mean and standard deviation of the number of photons that make it it to a 1 mm^2 detector?

$pN_0 = 0.05 \cdot 2.5\times10^{10} \cdot .050 \cdot (.1)^2 = 6.25\times10^5$ photons

mean = 6.25×10^5 photons
standard deviation = $\sqrt{6.25\times10^5}$ = 790 photons

Contrast and SNR for X-Rays

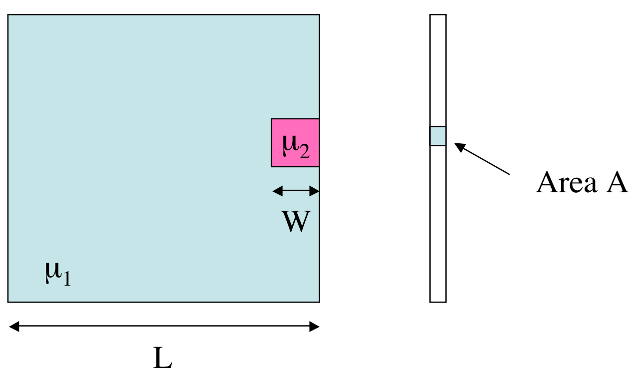
$$Contrast = C = \frac{\Delta I}{\bar{I}}$$

$$SNR = \frac{\Delta I}{\sigma_I}$$

$$= \frac{\text{Mean difference in \# of photons}}{\text{Standard Deviation of \# photons}}$$

$$= \frac{CpN_0}{\sqrt{pN_0}}$$

$$= C\sqrt{pN_0}$$



$$C = \frac{\Delta I}{\bar{I}} = \frac{N_0\left(\exp(-\mu_1 L) - \exp\left(-\left(\mu_1(L-W) + \mu_2 W\right)\right)\right)}{N_0 \exp(-\mu_1 L)}$$

$$SNR = \frac{CN_0 A \exp(-\mu_1 L)}{\sqrt{N_0 A \exp(-\mu_1 L)}} = C\sqrt{N_0 A \exp(-\mu_1 L)}$$

Signal to Noise Ratio for CT

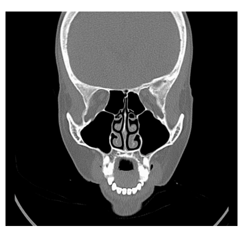
$$SNR = \frac{C\bar{\mu}}{\sigma_{\mu}}$$

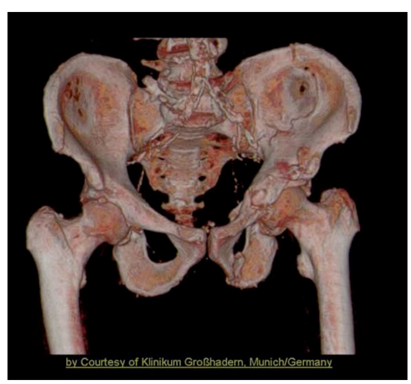
$$= \frac{C\bar{\mu}}{\sqrt{\frac{T}{M\bar{N}}\frac{2\pi^2\rho_0^3}{3}}}$$

$$\approx 0.4kC\bar{\mu}d^{3/2}\sqrt{M\bar{N}/T}$$

C = contrast
$\bar{\mu}$ = mean attenuation
$\bar{\mathrm{N}}$ = mean number of transmitted photon
T = spacing between detectors
M = number of views
ρ_0 = bandwidth of Ram-Lak filter ≈ k/d where d = width of detector
k = scaling constant, order unity

CT Applications





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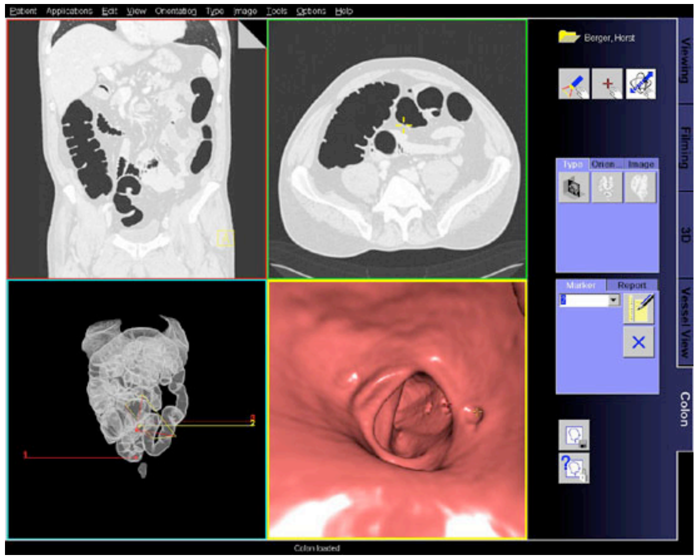
Virtual Colonoscopy



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