Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2010
CT Lecture 1

Computed Tomography

Scanner Generations

Figure S.12: Subsequent scanner generations: (a) first generation, (b) second generation, (c) third generation and (d) fourth generation CT scanner.
Single vs. Multi-slice

Suetens 2002

1G vs. 2G scanner

Example 6.1 from Prince and Links

Compare 1G vs. 2G scanner whose source - detector apparatus can move linearly at speed of 1 m/sec; FOV 0.5m; 360 projections over 180 degrees; 0.5 s for apparatus to rotate one angular increment, regardless of angle.

Question: Scan time for 1G scanner? Scan time for 2G scanner with 9 detectors spaced 0.5 degrees apart?

Answer:

1G scanner:

0.5m/(1m/s) = 0.5s per projection.
360 * 0.5 = 180s scan time.

2G scanner:

Required angular resolution is 180/360 = 0.5 degrees - agrees with spacing.
360/9 = 40 rotations required.
40 * 0.5 = 20s for rotation.
Total time = 40s.

Prince and Links 2005
3G, 6G, and 7G scanners

3G scanner: Typical scanner acquires 1000 projections with fanbeam angle of 30 to 60 degrees; 500 to 700 detectors; 1 to 20 seconds.

6G: Spiral/Helical CT
   - 60 cm torso scan: 30s
   - 24 cm lung scan: 12s
   - 15 cm angio: 30s

7G: Multislice CT
   - 64 or more parallel 1D projections.

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Detectors

CT Line Integral

\[ I_d = \int_0^{E_{\text{max}}} S_0(E) E \exp \left\{- \int_0^E \mu(s,E)ds \right\} dE \]

Monoenergetic Approximation

\[ I_d = I_0 \exp \left(- \int_0^E \mu(s,E)ds \right) \]

\[ g_d = -\log \left( \frac{I_d}{I_0} \right) \]

\[ = \int_0^E \mu(s,E)ds \]
CT Number

\[
CT\text{\_number} = \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \times 1000
\]

Measured in Hounsfield Units (HU)

Air: -1000 HU
Soft Tissue: -100 to 60 HU
Cortical Bones: 250 to 1000 HU
Metal and Contrast Agents: > 2000 HU

CT Display

Figure 5.4. CT-image of the chest with different window/level settings: (a) for the lungs (window 1000 and level -500) and (b) for the soft tissues (window 200 and level 50).

Direct Inverse Approach

\[
\begin{bmatrix}
\mu_1 \\
\mu_2 \\
\mu_3 \\
\mu_4
\end{bmatrix}
= 
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4
\end{bmatrix}
\]

4 equations, 4 unknowns.
Are these the correct equations to use?

\[
p_4 = p_1 + p_2 - p_3
\]

Matrix is not full rank.
Direct Inverse Approach

\[
\begin{array}{cccc}
\mu_1 & \mu_2 & p_1 & p_1 = \mu_1 + \mu_2 \\
\mu_3 & \mu_4 & p_2 & p_2 = \mu_3 + \mu_4 \\
& & p_3 & p_3 = \mu_1 + \mu_3 \\
& & p_4 & p_4 = \mu_1 + \mu_4 \\
& & p_5 & p_5 = \mu_1 + \mu_4 \\
\end{array}
\]

4 equations, 4 unknowns. These are linearly independent now.
In general for a NxN image, N^2 unknowns, N^2 equations.
This requires the inversion of a N^2 x N^2 matrix.
For a high-resolution 512x512 image, N^2 = 262144 equations.
Requires inversion of a 262144x262144 matrix!
Inversion process sensitive to measurement errors.

Iterative Inverse Approach

Algebraic Reconstruction Technique (ART)

\[
\begin{array}{ccc}
1 & 2 & 3 \\
3 & 4 & 7 \\
4 & 6 & 5 \\
\end{array} \rightarrow 
\begin{array}{cc}
2.5 & 2.5 \\
2.5 & 2.5 \\
5 & 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
1 & 2 & 3 \\
3 & 4 & 7 \\
5 & 5 & 5 \\
\end{array} \rightarrow 
\begin{array}{cc}
1.5 & 1.5 \\
3.5 & 3.5 \\
5 & 5 \\
\end{array}
\]

Backprojection

\[
\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{array} \rightarrow 
\begin{array}{ccc}
1 & 0 & 0 \\
1 & 2 & 1 \\
0 & 0 & 1 \\
\end{array} \rightarrow 
\begin{array}{ccc}
1 & 1 & 0 \\
1 & 3 & 1 \\
0 & 1 & 1 \\
\end{array} \rightarrow 
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 4 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

In-Class Exercise

\[
\begin{array}{cccc}
\begin{array}{cc}
\mu_1 & \mu_2 \\
5.7 & 11.3 \\
\end{array} \\
\begin{array}{cc}
\mu_3 & \mu_4 \\
8.2 & 8.8 \\
\end{array} \rightarrow 
\begin{array}{cc}
5.7 & 11.3 \\
8.2 & 8.8 \\
10.1 & 10.1 \\
\end{array}
\]

Suetens 2002
Projections

\[
\begin{bmatrix}
  r \\
  s \\
  x \\
  y
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

\[
I(r, \theta) = I_0 \exp \left( -\int_{L_{r,\theta}} \mu(x,y) ds \right)
\]

Radon Transform

\[
g(r, \theta) = \int_{-\infty}^{\infty} \mu(x(s), y(s)) ds
\]

\[
= \int_{-\infty}^{\infty} \mu(r \cos \theta - s \sin \theta, r \sin \theta + s \cos \theta) ds
\]

\[
g(r, \theta) = \int_{-\infty}^{\infty} \mu(x(s), y(s)) ds
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) \delta(x \cos \theta + y \sin \theta - r) dxdy
\]

\[
(x\hat{x} + y\hat{y}) \cdot (\cos \theta \hat{x} + \sin \theta \hat{y}) = r
\]

\[
x \cos \theta + y \sin \theta = r
\]
Example

\[ f(x,y) = \begin{cases} 
1 & x^2 + y^2 \leq 1 \\
0 & \text{otherwise} 
\end{cases} \]

\[ g(l, \theta = 0) = \int_{-\infty}^{\infty} f(l,y)dy = \int_{-|l|}^{0} dy = \begin{cases} 
2\sqrt{1-l^2} & |l| \leq 1 \\
0 & \text{otherwise} 
\end{cases} \]

Sinogram

Backprojection

\[ b(x_0, y) = p(l, \theta = 0) \Delta \theta = p(x_0, 0) \Delta \theta \]

\[ b_p(x_0, y) = g(x \cos \theta + y \sin \theta, \theta) \Delta \theta \]

\[ b(x, y) = B\{g(l, \theta)\} = \int_{0}^{\pi} p(x \cos \theta + y \sin \theta, \theta) d\theta \]
Backprojection

\[ b(x, y) = B\{P(l, \theta)\} = \int_0^\pi p(x \cos \theta + y \sin \theta, \theta) d\theta \]