

**HOMEWORK #7**  
**Due at the Beginning of Class on Thursday 12/02/10**

**Readings:**

Review Chapters 6 and 7 in Nishimura

**Problems:**

1. (*This problem was last year's final quiz*)

Consider the 2D object:  $f(x,y) = \cos(2\pi x - 2\pi\sqrt{3}y) \text{rect}(x/4) \text{rect}(\sqrt{3}y/4)$

- a) Sketch the object, labeling key features
  - b) Compute and sketch the Fourier Transform of the object, labeling key features.
  - c) Determine the values of  $\Delta k_x$  and  $\Delta k_y$  needed to avoid aliasing.
  - d) Assume that the desired resolution in the x and y directions is given by  $\frac{1}{2}$  cm and  $\frac{1}{2\sqrt{3}}$  cm. Determine  $k_{x,\max}$  and  $k_{y,\max}$  and also the number of voxels in the x and y directions.
  - e) Draw and label gradient waveforms that will provide sufficient coverage of k-space, according to the specifications in parts 3 and 4. Assume a spin-warp trajectory, a maximum available gradient of 4 G/cm, and a sampling rate of 10 micro-seconds. Where possible, minimize the duration of the gradients.
  - f) Assume that the voxels are arranged such that there is one voxel centered about the origin, so that there will be one more voxel along the negative directions than in the positive directions. Sketch the orientations of the spins along the x-axis (i.e.  $y=0$ ) for each of the sample points along the  $k_x$  axis (i.e.  $k_y = 0$ ). For what sample values of  $k_x$  does the absolute value of the vector sum of spins achieve a maximum or minimum? What are the vector sums at these points?
  - g) Now draw the spin orientations at the values of  $(k_x, k_y)$  that maximize the MRI signal. Provide a detailed interpretation of your answer.
  - h) Sketch the spin orientations for a value of  $(k_x, k_y)$  (assume  $k_x$  and  $k_y$  are both non-zero) for which the MRI signal is zero. Provide a detailed interpretation of your answer.
  - i) Use spin orientation diagrams to explain why there is aliasing when the sampling interval is doubled along the  $k_x$  axis. Provide a sketch of how the object appears when there is aliasing.
2. Problem 7.3 in Nishimura.
3. **MATLAB exercise 1.** In this exercise you will experiment with how different imaging parameters alter the contrast of an image. First download the file BE280A08\_hw6.mat from the course website. In this file you will find three matrices labeled csf, gm, and wm, which are the partial volume maps for cerebral spin fluid, gray matter, and white matter, respectively. Assume the values shown in the table below. Assume that your MRI system can have a minimum echo time (TE) of 3 ms and a maximum TR of 15,000ms. Finally, assume that you are using a saturation-recovery sequence. Come up with

sequence parameters that yield proton-density, T1-weighted, and T2-weighted images and use the partial volume maps to generate corresponding images. For the T1-weighted image, choose parameters that maximize the contrast between gray and white matter -- you will want to use MATLAB to search over possible T1 values.

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80

4. a) Design and sketch an RF and gradient pulse sequence to achieve an excitation pattern  $M_{xy}(x) = M_0 \cos(4\pi x)$ . This type of excitation pattern could be used to tag an object (e.g. the heart) with stripes. Assume that the RF sequence can play out Dirac delta functions, where the maximum available area of each delta function is  $6e-5$  Gauss-seconds and the maximum available gradient is  $4$  G/cm. Label your pulse sequence with the area and spacing of the delta functions and the amplitude and timings of the gradients. Choose the design that minimizes the spacing between the delta functions.
- b) Use MATLAB and the quiver function to plot out the orientation of the spins due to each of the delta functions as observed at the end of the pulse sequence. Show that the sum of the different spin profiles results in the desired excitation pattern.
- c) Next assume a more realistic case where you will achieve the excitation pattern with hard RF pulses, with maximum available RF amplitude of  $0.3$  Gauss. What are the RF pulse parameters that you would use to accomplish an excitation pattern that approximates  $M_{xy}(x) = M_0 \cos(4\pi x)$ . What is the actual excitation pattern that results when you use the hard RF pulses. Use MATLAB to plot and compare the two different excitation patterns.