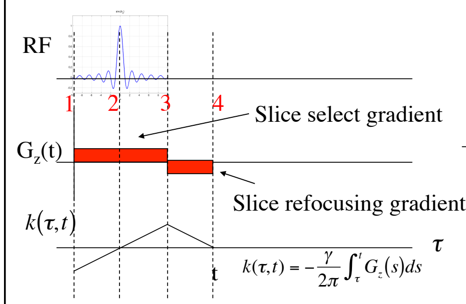
Bioengineering 280A
Principles of Biomedical Imaging

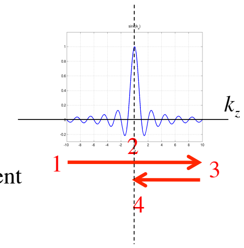
Fall Quarter 2010
MRI Lecture 5

Refocusing

$$M_{xy}(t,z) = jM_0 \int_{-\infty}^{t} \gamma B_1(\tau)\exp\left(j2\pi k(\tau,t)z\right)d\tau$$

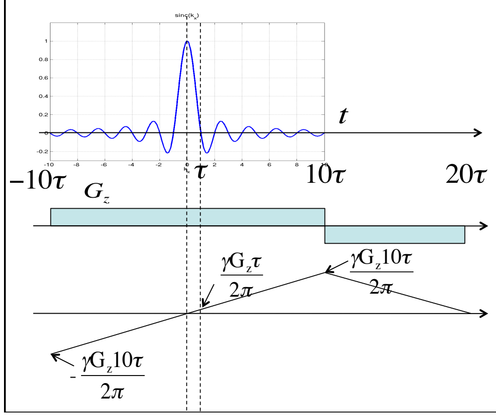
This has the form of an inverse Fourier transform, where we are integrating the contributions of the field $B_1(\tau)$ at the k - space point $k(\tau,t)$.





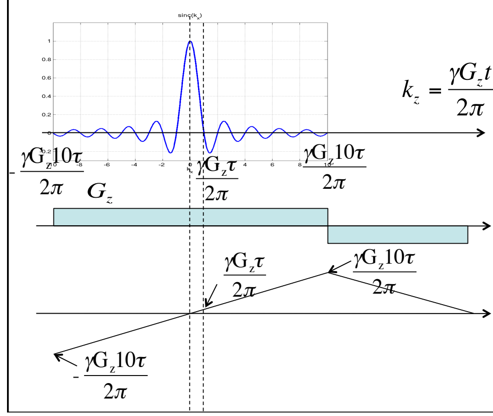
Small Tip Angle Example

$$B_1(t) = A\,\mathrm{sinc}(t/\tau)\,rect\left(\frac{t}{20\tau}\right)$$



Small Tip Angle Example

$$B_1(k_z) = A\,\mathrm{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right)$$



$$k_z = \frac{\gamma G_z t}{2\pi} \Rightarrow dk_z = \frac{\gamma G_z}{2\pi}dt$$

Small Tip Angle Example

$$B_1(k_z) = A\,\mathrm{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right)$$

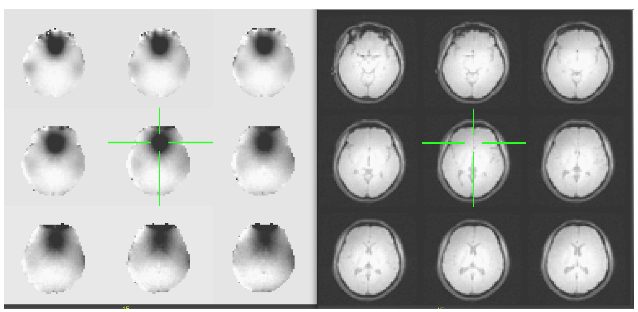
$$\begin{aligned}
M_{xy}(t,z) &= jM_0 \int_{-\infty}^{t} \gamma B_1(\tau)\exp\big(j2\pi k(\tau,t)z\big)d\tau \\
&= jM_0 \int_{-\infty}^{\infty} \gamma A\,\mathrm{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right)\exp\big(j2\pi k_z z\big)\frac{2\pi}{\gamma G_z}\,dk_z \\
&= jM_0 \gamma A \frac{2\pi}{\gamma G_z} F^{-1}\left(\mathrm{sinc}\left(\frac{k_z 2\pi}{\gamma G_z \tau}\right) rect\left(\frac{k_z 2\pi}{20\gamma G_z \tau}\right)\right) \\
&= jM_0 \gamma A \tau \cdot\, rect\left(\frac{\gamma G_z \tau}{2\pi} z\right) * \frac{20\gamma G_z \tau}{2\pi}\,\mathrm{sinc}\left(\frac{20\gamma G_z \tau}{2\pi} z\right)
\end{aligned}$$

Static Inhomogeneities

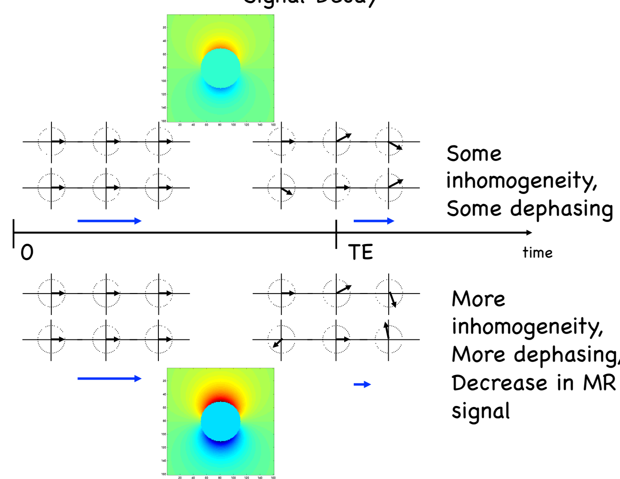
In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called "shim" coils, and the process of making the field more uniform is called "shimming". In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.

Field Inhomogeneities



Signal Decay



Static Inhomogeneities

The spatial nonuniformity in the field can be modeled by adding an additional term to our signal equation.

$$s_r(t) = \int_V M(\vec{r},t)dV$$
$$= \int_x \int_y \int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-j\omega_0 t}e^{-j\omega_E(\vec{r})t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz$$

The effect of this nonuniformity is to cause the spins to dephase with time and thus for the signal to decrease more rapidly. To first order this can be modeled as an additional decay term of the form

$$s_r(t) = \int_x \int_y \int_z M(x,y,z,0)e^{-t/T_2(\vec{r})}e^{-t/T_2'(\vec{r})}e^{-j\omega_0 t}\exp\left(-j\gamma\int_o^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right)dxdydz$$

T_2^* decay

The overall decay has the form.

$$\exp\left(-t/T_2^*(\vec{r})\right)$$

where

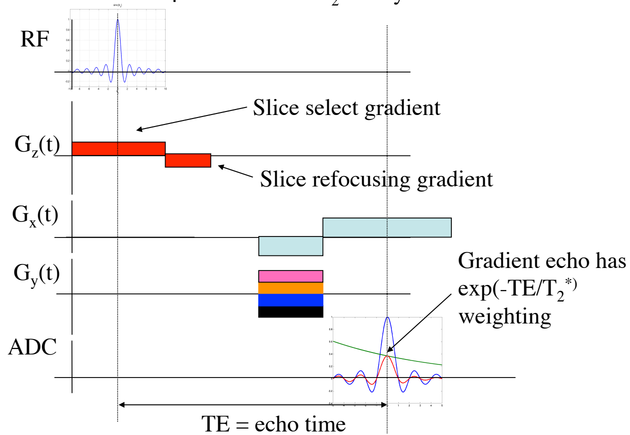
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.

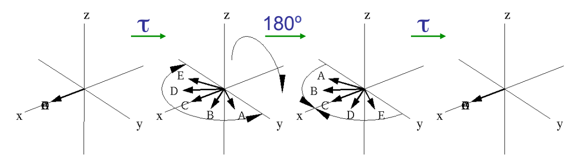
T_2^* decay

Gradient echo sequences exhibit T_2^* decay.



Spin Echo

Discovered by Erwin Hahn in 1950.

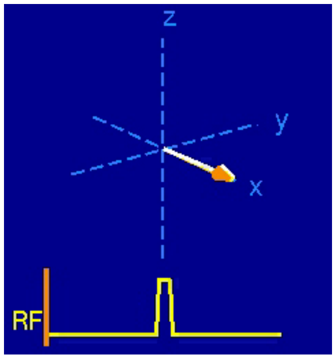


The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be T_2 dephasing due to random motion of spins.

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings. Erwin Hahn

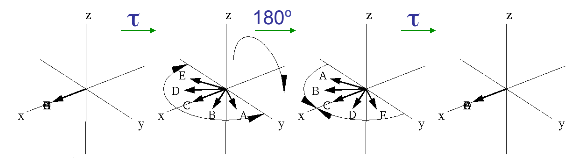
Image: Larry Frank

Spin Echo



Source: http://mrsrl.stanford.edu/~brian/mri-movies/

Spin Echo



Phase at time τ

$$\varphi(\tau) = \int_0^{\tau} -\omega_E(\vec{r})dt = -\omega_E(\vec{r})\tau$$

Phase after 180 pulse

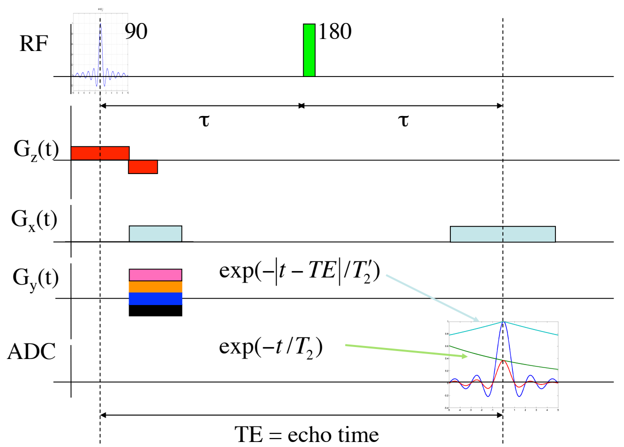
$$\varphi(\tau^{+}) = \omega_E(\vec{r})\tau$$

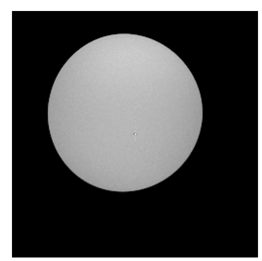
Phase at time 2τ

$$\varphi(2\tau) = -\omega_E(\vec{r})\tau + \omega_E(\vec{r})\tau = 0$$

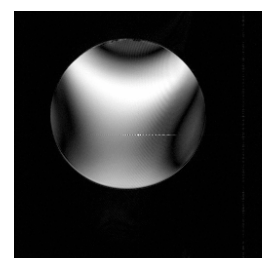
Image: Larry Frank

Spin Echo Pulse Sequence



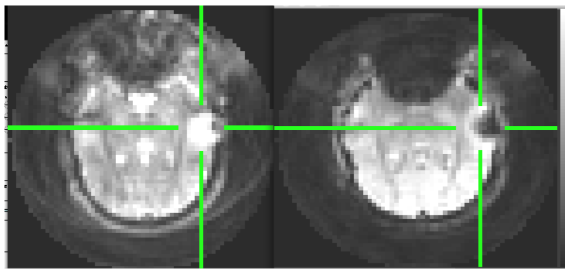


Spin-echo Image



Gradient-Echo Image

http://chickscope.beckman.uiuc.edu/roosts/carlartifacts.html

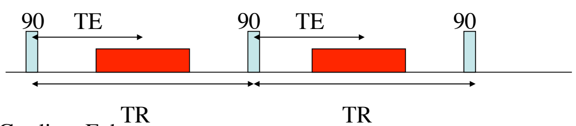


Spin-echo TE = 35 ms   Gradient Echo TE = 14ms

Image Contrast

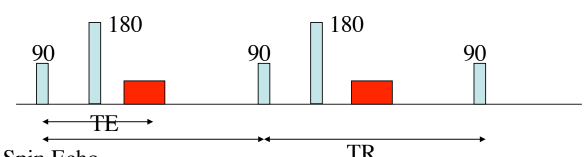
Different tissues exhibit different relaxation rates, T_1, T_2, and T_2^*. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).

Saturation Recovery Sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2^*(x,y)}$$



Spin Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]$$

T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.

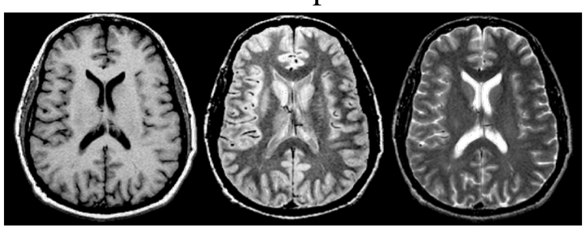
$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.

$$I(x,y) \approx \rho(x,y)$$

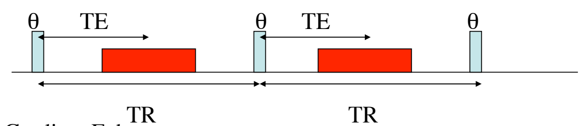
Example



T_1-weighted Density-weighted T_2-weighted

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80

FLASH sequence



Gradient Echo

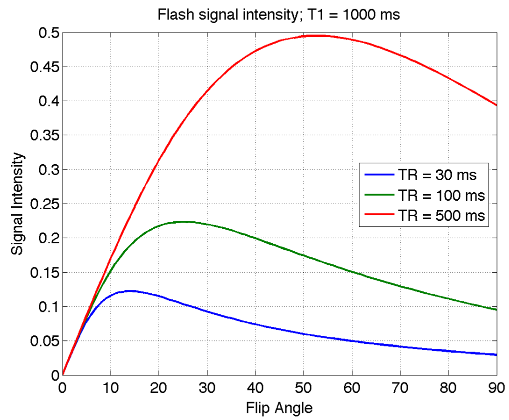
$$I(x,y) = \rho(x,y)\frac{\left[1-e^{-TR/T_1(x,y)}\right]\sin\theta}{\left[1-e^{-TR/T_1(x,y)}\cos\theta\right]}\exp(-TE/T_2^*)$$

Signal intensity is maximized at the Ernst Angle

$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

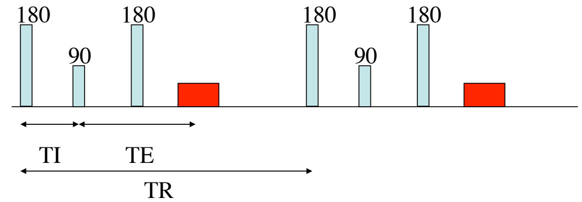
FLASH equation assumes no coherence from shot to shot. In practice this is achieved with RF spoiling.

FLASH sequence



$$\theta_E = \cos^{-1}\left(\exp(-TR/T_1)\right)$$

Inversion Recovery

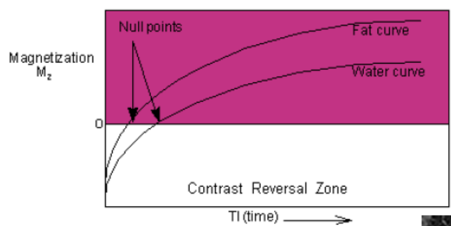


$$I(x,y) = \rho(x,y)\left[1 - 2e^{-TI/T_1(x,y)} + e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

Intensity is zero when inversion time is

$$TI = -T_1 \ln\left[\frac{1+\exp(-TR/T_1)}{2}\right]$$

Inversion Recovery



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