Moving Spins

So far we have assumed that the spins are not moving (aside from thermal motion giving rise to relaxation), and contrast has been based upon $T_1$, $T_2$, and proton density. We were able to achieve different contrasts by adjusting the appropriate pulse sequence parameters.

Biological samples are filled with moving spins, and we can also use MRI to image the movement. Examples: blood flow, diffusion of water in the white matter tracts. In addition, we can also sometimes induce motion into the object to image its mechanical properties, e.g. imaging of stress and strain with MR elastography.

Phase of Moving Spin

\[ \Delta B_z(x) = \frac{\gamma}{2\pi} \int_0^\tau \omega(t) dt \]

Phase of a Moving Spin

\[
\psi(t) = -\int_0^t \Delta \omega(\tau) d\tau \\
= -\int_0^t \gamma \Delta B(\tau) d\tau \\
= -\int_0^t \gamma \mathbf{G}(\tau) \cdot \mathbf{r}(\tau) d\tau \\
= -\gamma \int_0^t \left[ G_x(\tau) x(\tau) + G_y(\tau) y(\tau) + G_z(\tau) z(\tau) \right] d\tau
\]
Phase of Moving Spin

Consider motion along the x-axis

\[ x(t) = x_0 + vt + \frac{1}{2} at^2 \]

\[ q(t) = -\gamma \int_0^t G_x(\tau) x(\tau) d\tau \]

\[ = -\gamma \int_0^t G_x(\tau) \left[x_0 + vt + \frac{1}{2} a\tau^2\right] d\tau \]

\[ = -\gamma \left[x_0 \int_0^t G_x(\tau) d\tau + v \int_0^t G_x(\tau) \tau d\tau + \frac{a}{2} \int_0^t G_x(\tau) \tau^2 d\tau\right] \]

\[ = -\gamma \left[x_0 M_0 + v M_1 + \frac{a}{2} M_2\right] \]

Flow Moment Example

\[ M_0 = \int_0^T G_x(\tau) d\tau = 0 \]

\[ M_1 = \int_0^T G_x(\tau) \tau d\tau = -\int_0^T G_0 d\tau + \int_0^T G_0 \tau d\tau \]

\[ = G_0 \left[\frac{T^2}{2} + \frac{T^2}{2}\right] \]

\[ = G_0 \left[\frac{T^2}{2} + \frac{4T^2}{2} - \frac{T^2}{2}\right] = G_0 T^2 \]

Phase Contrast Angiography (PCA)

\[ \psi_1 = -\gamma v_x M_1 = \gamma v_x G_0 T^2 \]

\[ \psi_2 = -\gamma v_x M_1 = -\gamma v_x G_0 T^2 \]

\[ \Delta \psi = \psi_1 - \psi_2 = 2\gamma v_x G_0 T^2 \]

\[ v_x = \frac{\Delta \psi}{2G_0 T^2} \]
**PCA example**

- $G_0$

![Image of PCA example](http://www.medical.philips.com/main/products/mri/assets/images/case_of_week/cotw_51_s5.jpg)

**Aliasing in PCA**

Define VENC as the velocity at which the phase is 180 degrees.

$$VENC = \frac{\pi}{\gamma G_0 T^2}$$

Because of phase wrapping, the velocity of spins flowing faster than VENC is ambiguous.

**Aliasing Solutions**

- Use data from regions with slower flow.
- Use multiple VENC values so that the phase differences are smaller than $\pi$ radians.

$$\phi_1 = \frac{v_1}{VENC_1}$$
$$\phi_2 = \frac{v_2}{VENC_2}$$
$$\phi_1 - \phi_2 = \pi v \left( \frac{1}{VENC_1} - \frac{1}{VENC_2} \right)$$

**Flow Artifacts**

During readout moving spins within the object accumulate phase that is in addition to the phase used for imaging. This leads to:

1) Net phase at echo time $TE = 2T$.
2) An apparent shift in position of the object.
3) Blurring of the object due to a quadratic phase term.
Flow Artifacts

Plug Flow

Laminar Flow

All moving spins in the voxel experience the same phase shift at echo time.

Spins have different phase shifts at echo time. The dephasing causes the cancelation and signal dropout.

Flow Compensation

Readout Gradient

Echo Time $TE$

$G_0$

$-2G_0$

At $TE$ both the first and second order moments are zero, so both stationary and moving spins have zero net phase.

Inflow Effect

Prior to imaging

Relaxed spins flowing in

Saturated spins

Time of Flight Angiography
Cerebral Blood Flow (CBF)

CBF = Perfusion
= Rate of delivery of arterial blood to a capillary bed in tissue.

Units: \( \frac{\text{ml of Blood}}{(100 \text{ grams of tissue})(\text{minute})} \)

Typical value is 60 ml(100g-min) or 60 ml(100 ml-min) = 0.01 s\(^{-1}\), assuming average density of brain equals 1 gm/ml

Arterial spin labeling (ASL)
1: Tag by Magnetic Inversion
2: Control - Tag = \( \Delta M \propto \text{CBF} \)

Bereczki et al 1992
ASL Signal Equation

\[ \Delta M = CBF \cdot A_{\text{eff}} \]

\( A_{\text{eff}} \) is the effective area of the arterial bolus. It depends on both physiology and pulse sequence parameters.

Multislice CASL and PICORE

CASL

PICORE

QUIPSS II

Credit: E. Wong
**Diffusion**

\[ \langle \Delta x^2 \rangle = Nd^2 = 2DT \]

D = diffusivity

In brain:

\[ D \approx 0.001 \text{ mm}^2/\text{s} \]

For T=100 msec,

\[ \Delta x \approx 15 \mu \text{m} \]

**Diffusing Spins**

\[ \Delta B_z(x) \]

\[ \Delta B_z(x) \]

\[ x \]

\[ \text{time} \]

**Diffusion Weighting**

\[ S \propto e^{-\gamma^2 G_0 \delta^2 DT} = e^{-bd} \]

where \( b = \gamma^2 G_0^2 \delta^2 (T - \delta/3) \)

**Diffusion Weighted Images**

T2 weighted Diffusion Weighted Angiogram

After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

http://lehighmri.com/cases/dwi/patient-b.html
Restricted Diffusion

D depends on direction

Diffusion tensor:
- 3 values of D
- 3 angles

Credit: Larry Frank

Diffusion Imaging Example

Q-ball imaging

Tuch et al, Neuron 2003

Fiber tract mapping of neural connectivity

Courtesy of L. Frank