BENG280A, Principles of Biomedical Imaging 10/17/12
Fall Quarter 2012

HOMEWORK #3
Due at 5 pm on Wednesday 10/24/12

Homework policy: Homeworks can be turned in during class or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material.

Readings: Review sections 5.1-5.5 and 5.6.2. Read Section 5.7.

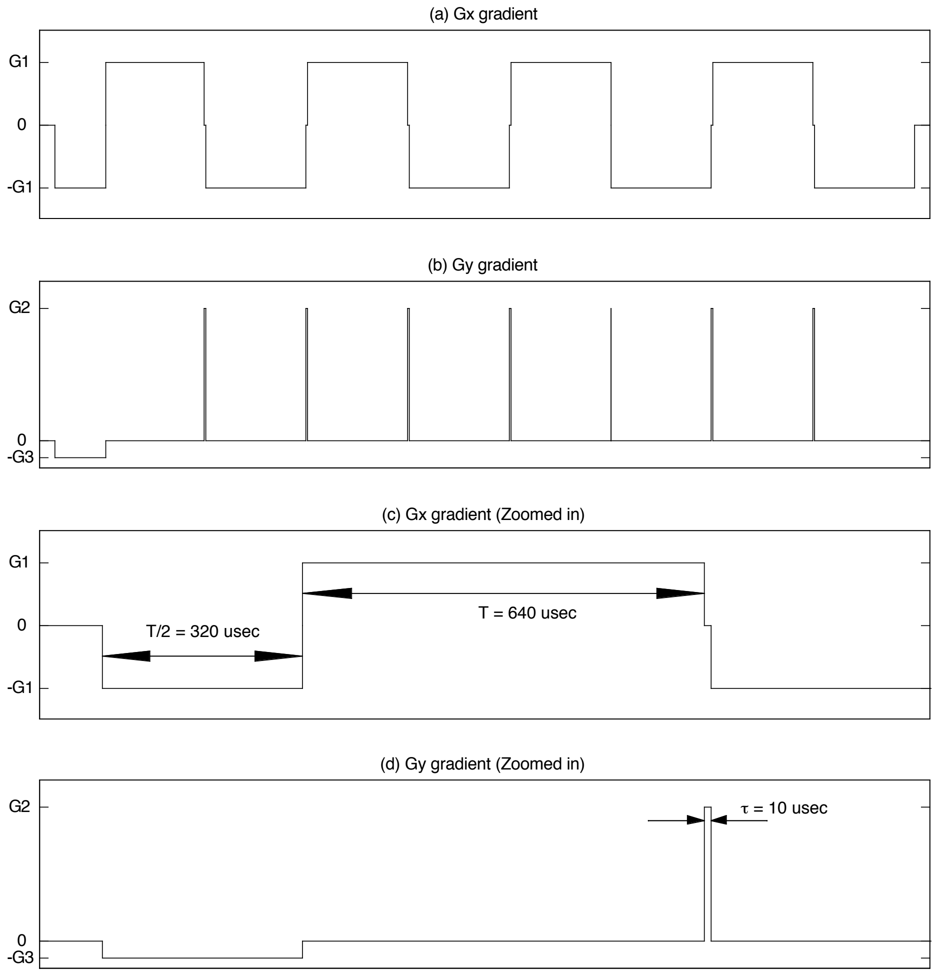
Problem 1. On last week's HW you used the Fourier shift theorem to derive the transform of the object $m(x,y) = rect(x+1, y+1) - rect(x-1, y-1)$. This week, you will use the convolution theorem to derive the transform. First, express the object as the convolution of a rect and the appropriate shifted delta functions. Then derive the transform by making use of the theorem and the expressions for the transform of a rect and of the shifted delta functions. Compare your answer to the one you obtained for HW2.

Problem 2
Consider the 2D object: $f(x,y) = \cos(2\pi x - 2\pi\sqrt{3}y)rect(x/4)rect(\sqrt{3}y/4)$
a) Sketch the object, labeling key features
b) Compute and sketch the Fourier Transform of the object, labeling key features.

Problem 3 (this is Problem 12.1 in Prince and Links)
A non-uniform magnetic field pointing in the z-direction is applied to a sample of protons. The B-field (in Tesla) varies as a function of z (in cm) as follows: B(z) = 1+0.5z. The magnetization vector M precesses around the z-axis. Suppose at time t = 0, all magnetization vectors have the same phase. At what time will the magnetization vector at z = 1 cm and z = 0 have the same phase again?

Problem 4
Consider the gradient waveforms shown in the figure on the next page. The full waveforms are shown in panels (a) and (b), and zoomed-in views are shown in (c) and (d). The analog-to-digital converter (ADC) is turned on during the flat parts of the readout (Gx) gradients with a sampling rate of Δt.
(a) Determine the sequence parameters (G1, G2 and G3, and Δt) to achieve the following image specifications: FOV_x = 192mm; FOV_y = 192 mm, $\delta_x = 3$ mm and $\delta_y = 24$ mm.
(b) Draw the k-space trajectory (make sure to label units correctly).

MATLAB exercise. In this exercise you will generate figures similar to those in Figure 5.7. Consider a uniform object with FOV_x = 4cm; and FOV_y= 2cm. The desired resolution is 0.5cm in both the x and y directions. Define a grid for the simulation of the form [x,y]=meshgrid([-2:dx:2],[-1:dx:1]);dx = .1. Use the **quiver** command in MATLAB to plot out the relative phases of the spins for k_x ranging from $-W_{k_x}/2$ to $W_{k_x}/2$ in steps of Δk_x. You may assume that $k_y = 0$ for your simulations. Verify that the periods of spatial variation of the spin phases are what you expect. Also, comment on the differences in spatial variation between the negative and positive frequencies. **Hint:** Define a complex exponential and then use the real and imaginary parts as inputs into the quiver command. You may also want to play around with the scaling parameter in the quiver function to adjust the length of the arrows.


(a) Gx gradient
G1
0
-G1
(b) Gy gradient
G2
0
-G3
(c) Gx gradient (Zoomed in)
G1
0
-G1
T/2 = 320 usec
T = 640 usec
(d) Gy gradient (Zoomed in)
G2
0
-G3
τ = 10 usec