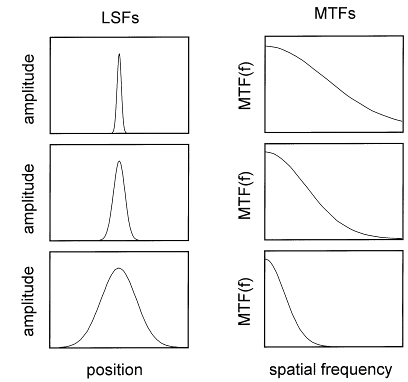
Bioengineering 280A
Principles of Biomedical Imaging

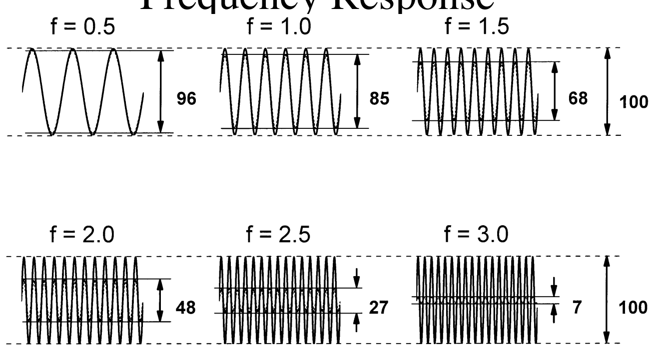
Fall Quarter 2012
MRI Lecture 4

MTF = Fourier Transform of Impulse Response



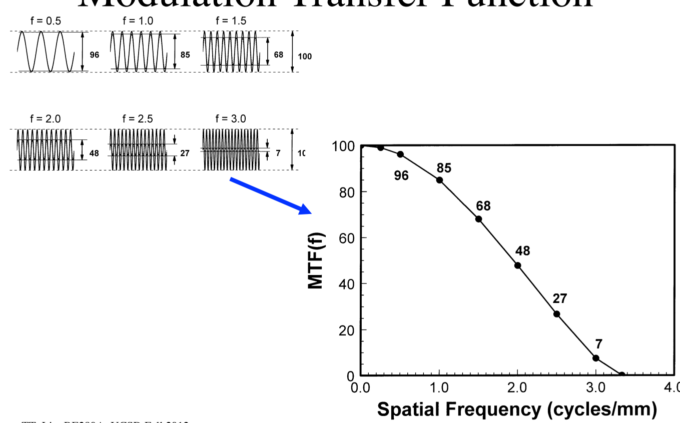
Bushberg et al 2001

Modulation Transfer Function (MTF) or Frequency Response



Bushberg et al 2001

Modulation Transfer Function



Bushberg et al 2001

Figure 1:

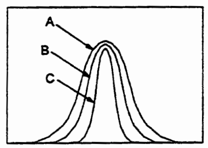
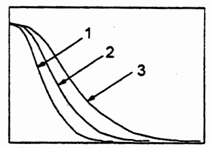


Figure 2:



8. Referring to Figure 1 (above) which demonstrates 3 different line spread functions (LSF), which LSF will yield the best spatial resolution?

10. Referring to Figure 1 which shows LSFs, and Figure 2 which shows the corresponding modulation transfer functions (MTFs), which MTF corresponds to LSF C?

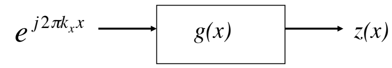
A. MTF number 1
B. MTF number 2
C. MTF number 3

D74. The intrinsic resolution of a gamma camera is 5 mm. The collimator resolution is 10 mm. The overall system resolution is ______ mm.

A. 15
B. 11.2
C. 7.5
D. 5.0
E. 0.5

Eigenfunctions

The fundamental nature of the convolution theorem may be better understood by observing that the complex exponentials are eigenfunctions of the convolution operator.

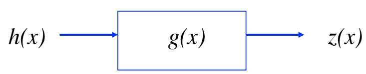


$$\begin{aligned} z(x) &= g(x) * e^{j2\pi k_x x} \\ &= \int_{-\infty}^{\infty} g(u) e^{j2\pi k_x (x-u)} du \\ &= G(k_x) e^{j2\pi k_x x} \end{aligned}$$

The response of a linear shift invariant system to a complex exponential is simply the exponential multiplied by the FT of the system's impulse response.

Convolution/Multiplication

Now consider an arbitrary input *h(x)*.



Recall that we can express *h(x)* as the integral of weighted complex exponentials.

$$h(x) = \int_{-\infty}^{\infty} H(k_x) e^{j2\pi k_x x} dk_x$$

Each of these exponentials is weighted by $G(k_x)$ so that the response may be written as

$$z(x) = \int_{-\infty}^{\infty} G(k_x) H(k_x) e^{j2\pi k_x x} dk_x$$

Convolution/Modulation Theorem

$$\begin{aligned} F\{g(x) * h(x)\} &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} g(u) * h(x-u) du \right] e^{-j2\pi k_x x} dx \\ &= \int_{-\infty}^{\infty} g(u) \int_{-\infty}^{\infty} h(x-u) e^{-j2\pi k_x x} dx du \\ &= \int_{-\infty}^{\infty} g(u) H(k_x) e^{-j2\pi k_x u} du \\ &= G(k_x) H(k_x) \end{aligned}$$

Convolution in the spatial domain transforms into multiplication in the frequency domain. Dual is modulation

$$F\{g(x)h(x)\} = G(k_x) * H(k_x)$$

2D Convolution/Multiplication

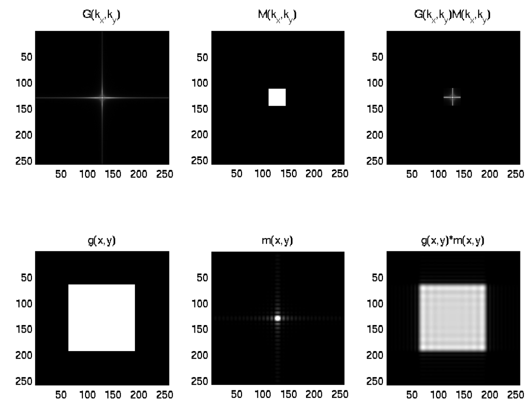
Convolution

$$F\left[g(x,y) ** h(x,y)\right] = G(k_x,k_y)H(k_x,k_y)$$

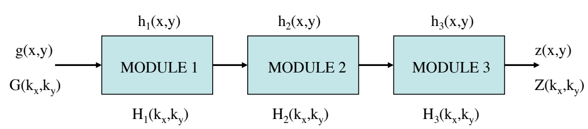
Multiplication

$$F\left[g(x,y)h(x,y)\right] = G(k_x,k_y) ** H(k_x,k_y)$$

Convolution Example



Response of an Imaging System

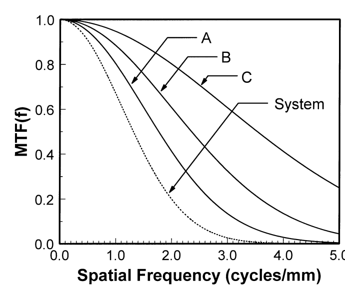


$$z(x,y)=g(x,y)**h_1(x,y)**h_2(x,y)**h_3(x,y)$$

$$Z(k_x,k_y)=G(k_x,k_y)\ H_1(k_x,k_y)\ H_2(k_x,k_y)\ H_3(k_x,k_y)$$

TT. Liu, BE280A, UCSD Fall 2012

System MTF = Product of MTFs of Components



Bushberg et al 2001

TT. Liu, BE280A, UCSD Fall 2012

Useful Approximation

$$FWHM_{System} = \sqrt{FWHM_1^2 + FWHM_2^2 + \cdots FWHM_N^2}$$

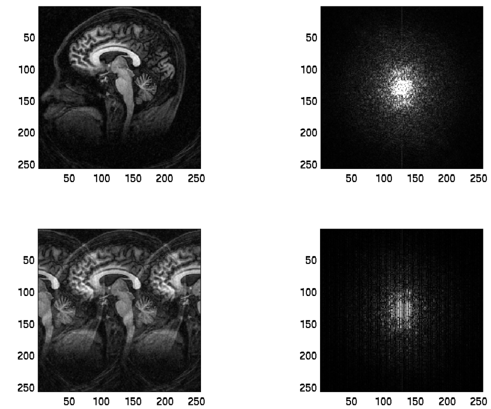
Example

$$FWHM_1 = 1mm$$

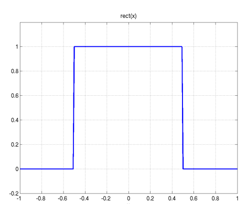
$$FWHM_2 = 2mm$$

$$FWHM_{system} = \sqrt{5} = 2.24mm$$

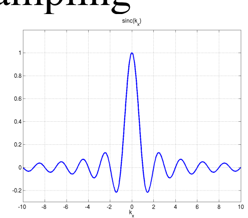
Sampling in k-space



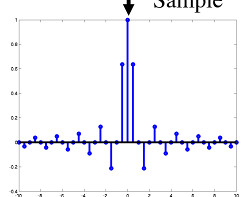
Fourier Sampling



F



Sample

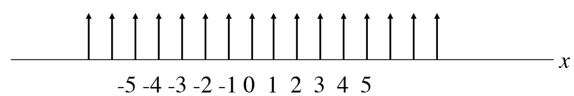


Instead of sampling the signal, we sample its Fourier Transform

??? F^{-1}

Comb Function

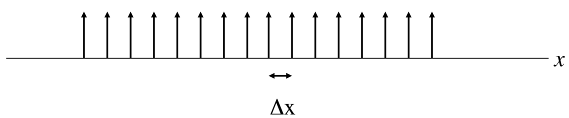
$$comb(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$$



Other names: Impulse train, bed of nails, shah function.

Scaled Comb Function

$$comb\left(\frac{x}{\Delta x}\right) = \sum_{n=-\infty}^{\infty} \delta(\frac{x}{\Delta x} - n)$$
$$= \sum_{n=-\infty}^{\infty} \delta(\frac{x - n\Delta x}{\Delta x})$$
$$= \Delta x \sum_{n=-\infty}^{\infty} \delta(x - n\Delta x)$$

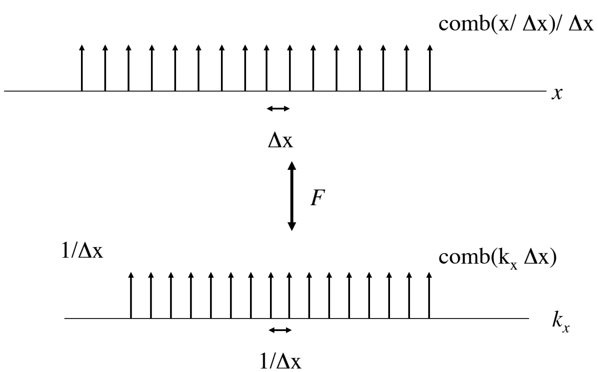


Fourier Transform of comb(x)

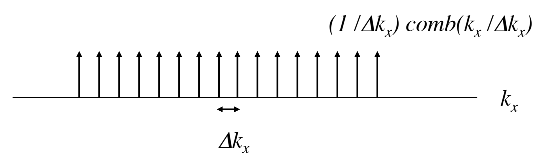
$$F\left[comb(x)\right] = comb(k_x)$$
$$= \sum_{n=-\infty}^{\infty} \delta(k_x - n)$$

$$F\left[\frac{1}{\Delta x} comb(\frac{x}{\Delta x})\right] = \frac{1}{\Delta x} \Delta x comb(k_x \Delta x)$$
$$= \sum_{n=-\infty}^{\infty} \delta(k_x \Delta x - n)$$
$$= \frac{1}{\Delta x} \sum_{n=-\infty}^{\infty} \delta(k_x - \frac{n}{\Delta x})$$

Fourier Transform of comb(x/ Δx)

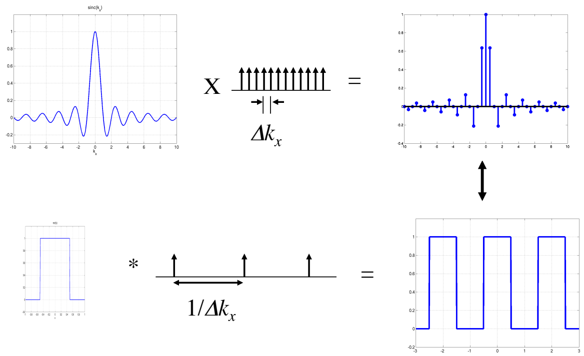


Fourier Sampling

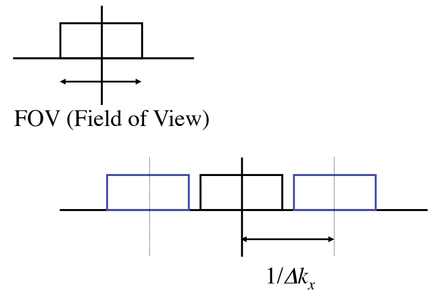


$$G_S(k_x) = G(k_x) \frac{1}{\Delta k_x} comb\left(\frac{k_x}{\Delta k_x}\right)$$
$$= G(k_x) \sum_{n=-\infty}^{\infty} \delta(k_x - n\Delta k_x)$$
$$= \sum_{n=-\infty}^{\infty} G(n\Delta k_x) \delta(k_x - n\Delta k_x)$$

Fourier Sampling -- Inverse Transform

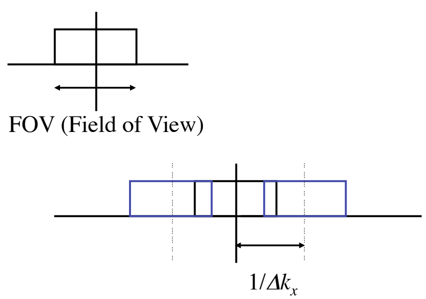


Nyquist Condition



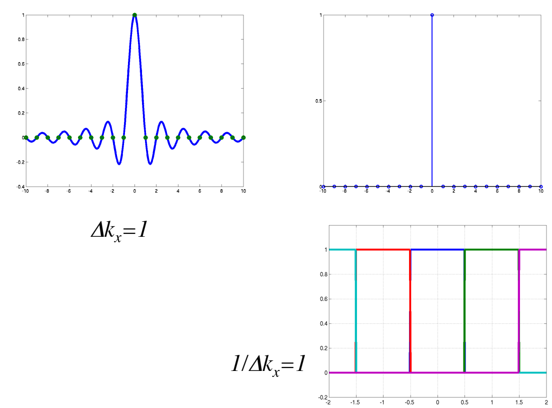
To avoid overlap, $1/\Delta k_x$> FOV, or equivalently, Δk_x<1/FOV

Aliasing



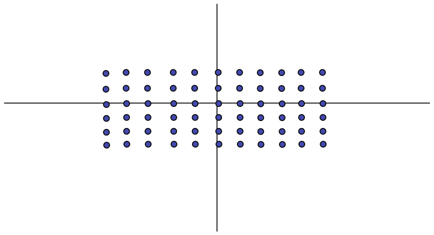
Aliasing occurs when $1/\Delta k_x$< FOV

Aliasing Example



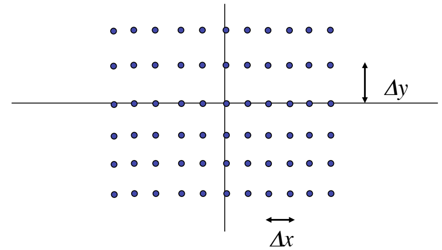
2D Comb Function

$$comb(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m,y-n)$$
$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m)\delta(y-n)$$
$$= comb(x)comb(y)$$

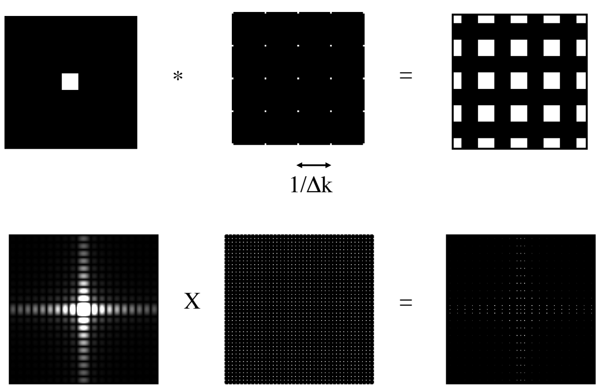


Scaled 2D Comb Function

$$comb(x/\Delta x, y/\Delta y) = comb(x/\Delta x)comb(y/\Delta y)$$
$$= \Delta x\Delta y \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-m\Delta x)\delta(y-n\Delta y)$$



TT. Liu, BE280A, UCSD Fall 2012



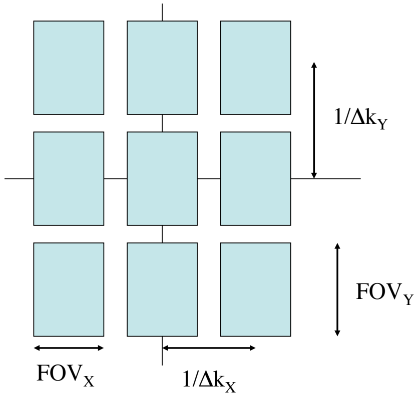
TT. Liu, BE280A, UCSD Fall 2012

2D k-space sampling

$$G_S(k_x,k_y) = G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right)$$
$$= G(k_x,k_y)\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(k_x - m\Delta k_x, k_y - n\Delta k_y)$$
$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} G(m\Delta k_x, n\Delta k_y)\delta(k_x - m\Delta k_x, k_y - n\Delta k_y)$$

TT. Liu, BE280A, UCSD Fall 2012

Nyquist Conditions



$1/\Delta k_Y > FOV_Y$

$1/\Delta k_X > FOV_X$

Windowing

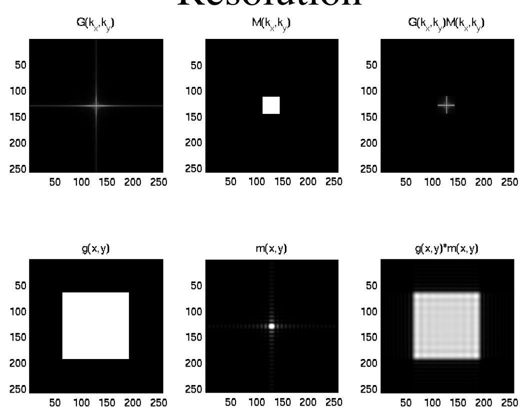
Windowing the data in Fourier space

$$G_W(k_x,k_y) = G(k_x,k_y)W(k_x,k_y)$$

Results in convolution of the object with the inverse transform of the window

$$g_W(x,y) = g(x,y) * w(x,y)$$

Resolution



Windowing Example

$$W(k_x,k_y) = rect\left(\frac{k_x}{W_{k_x}}\right)rect\left(\frac{k_y}{W_{k_y}}\right)$$

$$w(x,y) = F^{-1}\left[rect\left(\frac{k_x}{W_{k_x}}\right)rect\left(\frac{k_y}{W_{k_y}}\right)\right]$$
$$= W_{k_x}W_{k_y}\,\mathrm{sinc}(W_{k_x}x)\mathrm{sinc}(W_{k_y}y)$$

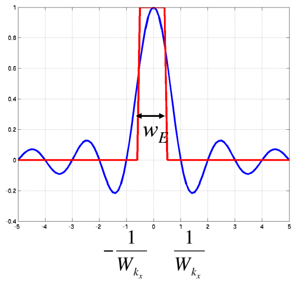
$$g_W(x,y) = g(x,y) ** W_{k_x}W_{k_y}\,\mathrm{sinc}(W_{k_x}x)\mathrm{sinc}(W_{k_y}y)$$

Effective Width

$$w_E = \frac{1}{w(0)} \int_{-\infty}^{\infty} w(x)dx$$

Example

$$w_E = \frac{1}{1}\int_{-\infty}^{\infty} \text{sinc}(W_{k_x} x)dx$$
$$= F\left[\text{sinc}(W_{k_x} x)\right]\Big|_{k_x=0}$$
$$= \frac{1}{W_{k_x}} rect\left(\frac{k_x}{W_{k_x}}\right)\Bigg|_{k_x=0}$$
$$= \frac{1}{W_{k_x}}$$

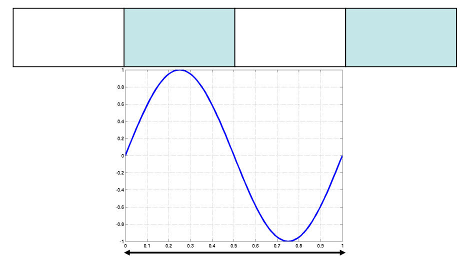


Resolution and spatial frequency

With a window of width W_{k_x} the highest spatial frequency is $W_{k_x}/2$.

This corresponds to a spatial period of $2/W_{k_x}$.

$\frac{1}{W_{k_x}}$ = Effective Width

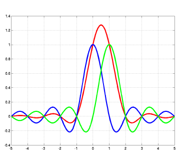


$\frac{2}{W_{k_x}}$

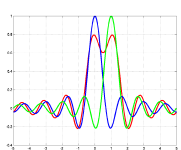
Windowing Example

$$g(x,y) = [\delta(x) + \delta(x-1)]\delta(y)$$

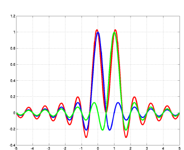
$$g_W(x,y) = [\delta(x) + \delta(x-1)]\delta(y) ** W_{k_x} W_{k_y} \text{sinc}(W_{k_x} x)\text{sinc}(W_{k_y} y)$$
$$= W_{k_x} W_{k_y}\left([\delta(x) + \delta(x-1)] * \text{sinc}(W_{k_x} x)\right)\text{sinc}(W_{k_y} y)$$
$$= W_{k_x} W_{k_y}\left(\text{sinc}(W_{k_x} x) + \text{sinc}(W_{k_x}(x-1))\right)\text{sinc}(W_{k_y} y)$$



$W_{k_x} = 1$

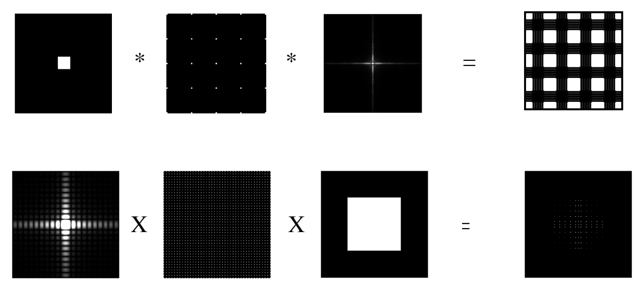


$W_{k_x} = 1.5$



$W_{k_x} = 2$

Sampling and Windowing



Sampling and Windowing

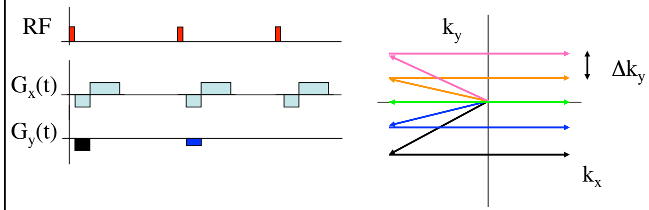
Sampling and windowing the data in Fourier space

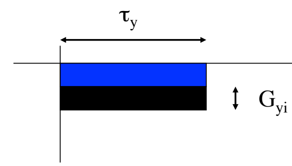
$$G_{SW}(k_x,k_y) = G(k_x,k_y)\frac{1}{\Delta k_x \Delta k_y} comb\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right) rect\left(\frac{k_x}{W_{k_x}},\frac{k_y}{W_{k_y}}\right)$$

Results in replication and convolution in object space.

$$g_{SW}(x,y) = W_{k_x} W_{ky} g(x,y) ** comb(\Delta k_x x, \Delta k_y y) ** \mathrm{sinc}(W_{k_x} x)\,\mathrm{sinc}(W_{k_y} y)$$

Sampling in k_y



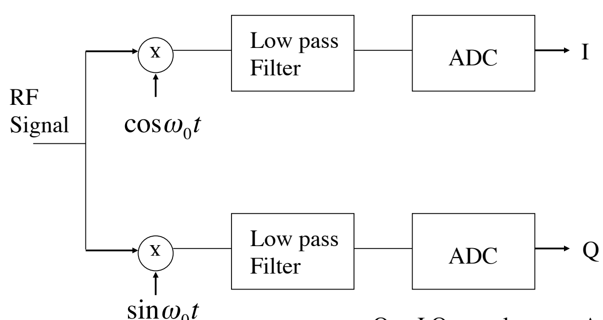


$$\Delta k_y = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

$$FOV_y = \frac{1}{\Delta k_y}$$

TT. Liu, BE280A, UCSD Fall 2012

Sampling in k_x



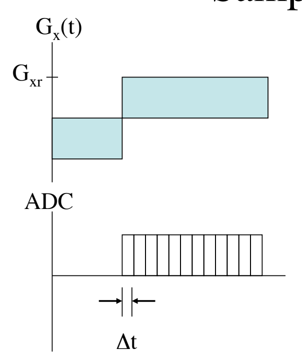
One I,Q sample every Δt

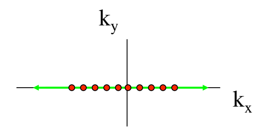
M= I+jQ

Note: In practice, there are number of ways of implementing this processing.

TT. Liu, BE280A, UCSD Fall 2012

Sampling in k_x





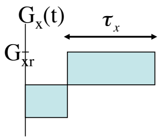
$$\Delta k_x = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

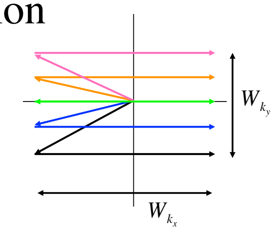
$$FOV_x = \frac{1}{\Delta k_x}$$

TT. Liu, BE280A, UCSD Fall 2012

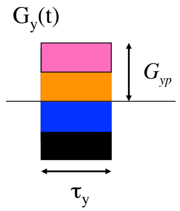
Resolution

$$\delta_x = \frac{1}{W_{k_x}} = \frac{1}{2k_{x,\max}} = \frac{1}{\frac{\gamma}{2\pi} G_{xr}\tau_x}$$





$$\delta_y = \frac{1}{W_{k_y}} = \frac{1}{2k_{y,\max}} = \frac{1}{\frac{\gamma}{2\pi} 2G_{yp}\tau_y}$$



Example

Goal:

$FOV_x = FOV_y = 25.6$ cm

$\delta_x = \delta_y = 0.1$ cm

Readout Gradient:

$$FOV_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr}\Delta t}$$

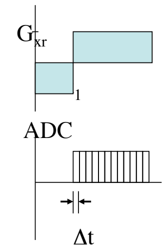
Pick $\Delta t = 32\ \mu$sec

$$G_{xr} = \frac{1}{FOV_x \frac{\gamma}{2\pi}\Delta t} = \frac{1}{(25.6cm)(42.57\times 10^6 T^{-1}s^{-1})(32\times 10^{-6} s)}$$

$= 2.8675 \times 10^{-5}$ T/cm

$= .28675$ G/cm

1 Gauss $= 1\times 10^{-4}$ Tesla



Example

Readout Gradient:

$$\delta_x = \frac{1}{\frac{\gamma}{2\pi} G_{xr}\tau_x}$$

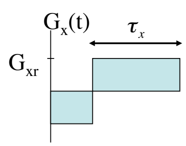
$$\tau_x = \frac{1}{\delta_x \frac{\gamma}{2\pi} G_{xr}} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(0.28675\ \text{G/cm})}$$

$= 8.192$ ms

$= N_{read}\Delta t$

where

$$N_{read} = \frac{FOV_x}{\delta_x} = 256$$



Example

Phase - Encode Gradient:

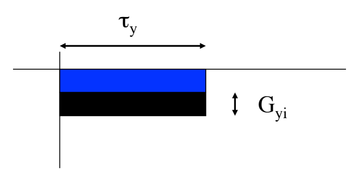
$$FOV_y = \frac{1}{\frac{\gamma}{2\pi} G_{yi}\tau_y}$$

Pick $\tau_y = 4.096$ msec

$$G_{yi} = \frac{1}{FOV_y \frac{\gamma}{2\pi}\tau_y} = \frac{1}{(25.6cm)(42.57\times 10^6 T^{-1}s^{-1})(4.096\times 10^{-3} s)}$$

$= 2.2402\times 10^{-7}$ T/cm

$= .00224$ G/cm



Example

Phase - Encode Gradient :

$$\delta_y = \frac{1}{\frac{\gamma}{2\pi} 2 G_{yp} \tau_y}$$

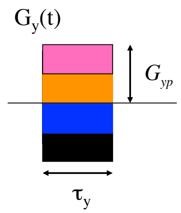
$$G_{yp} = \frac{1}{\delta_y 2 \frac{\gamma}{2\pi} \tau_y} = \frac{1}{(0.1cm)(4257\ G^{-1}s^{-1})(4.096 \times 10^{-3}\ s)}$$

$$= 0.2868 \text{ G/cm}$$

$$= \frac{N_p}{2} G_{yi}$$

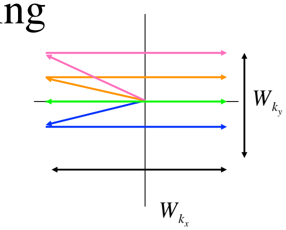
where

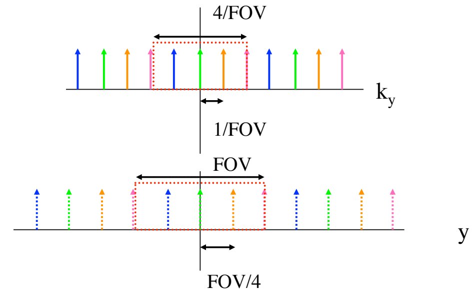
$$N_p = \frac{FOV_y}{\delta_y} = 256$$



Sampling

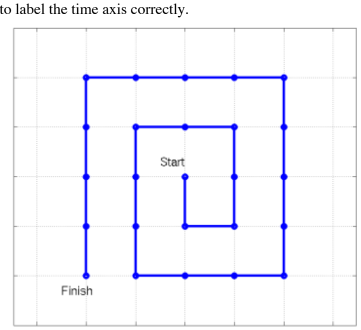
In practice, an even number (typically power of 2) sample is usually taken in each direction to take advantage of the Fast Fourier Transform (FFT) for reconstruction.

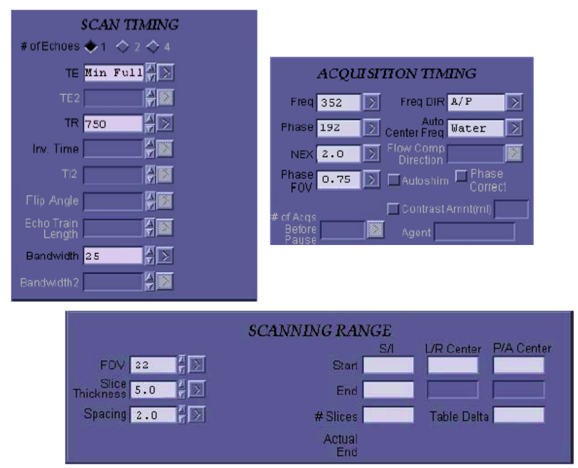




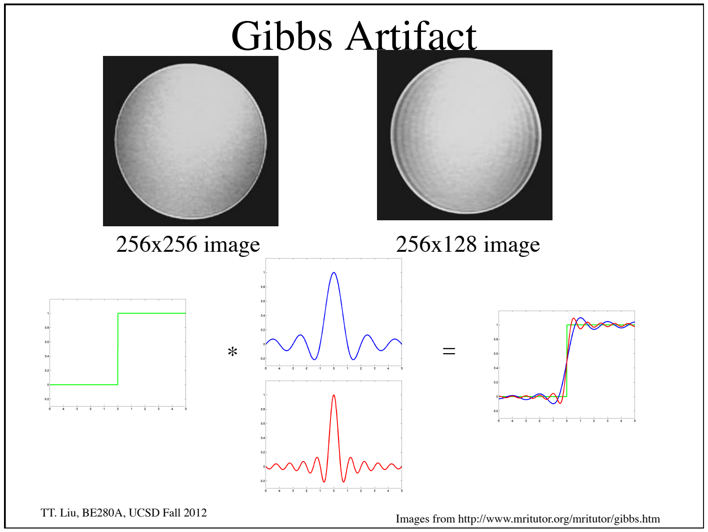
Example

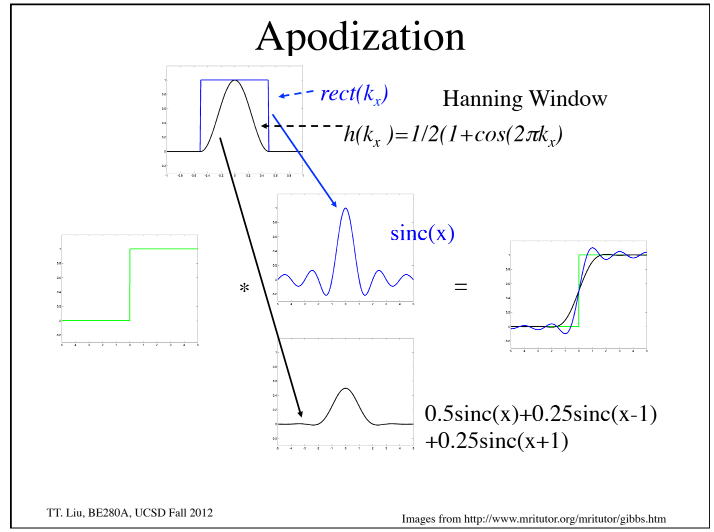
Consider the k-space trajectory shown below. ADC samples are acquired at the points shown with $\Delta t = 10\ \mu sec$. The desired FOV (both x and y) is 10 cm and the desired resolution (both x and y) is 2.5 cm. Draw the gradient waveforms required to achieve the k-space trajectory. Label the waveform with the gradient amplitudes required to achieve the desired FOV and resolution. Also, make sure to label the time axis correctly.

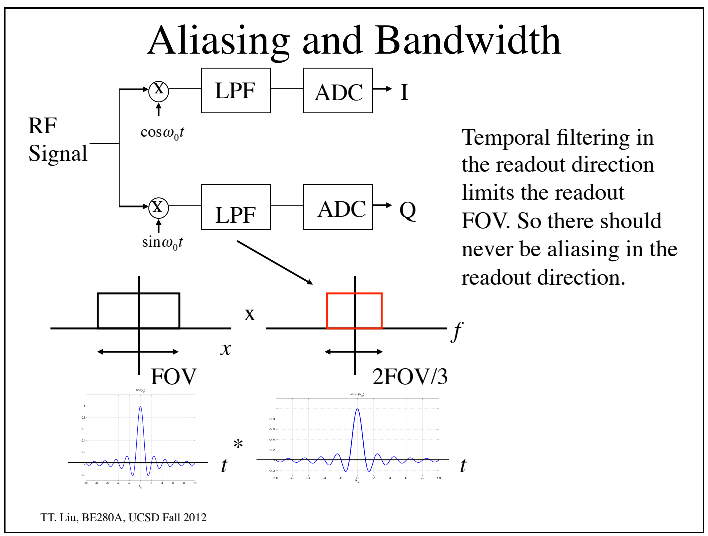




GE Medical Systems 2003


Gibbs Artifact
256x256 image
256x128 image
*
=
TT. Liu, BE280A, UCSD Fall 2012
Images from http://www.mritutor.org/mritutor/gibbs.htm


Apodization
rect(k_x)
Hanning Window
h(k_x)=1/2(1+cos(2πk_x)
sinc(x)
*
=
0.5sinc(x)+0.25sinc(x-1)
+0.25sinc(x+1)
TT. Liu, BE280A, UCSD Fall 2012
Images from http://www.mritutor.org/mritutor/gibbs.htm


Aliasing and Bandwidth
RF
Signal
cosω0t
sinω0t
LPF
ADC
I
LPF
ADC
Q
Temporal filtering in the readout direction limits the readout FOV. So there should never be aliasing in the readout direction.
x
x
FOV
f
2FOV/3
t
*
t
TT. Liu, BE280A, UCSD Fall 2012

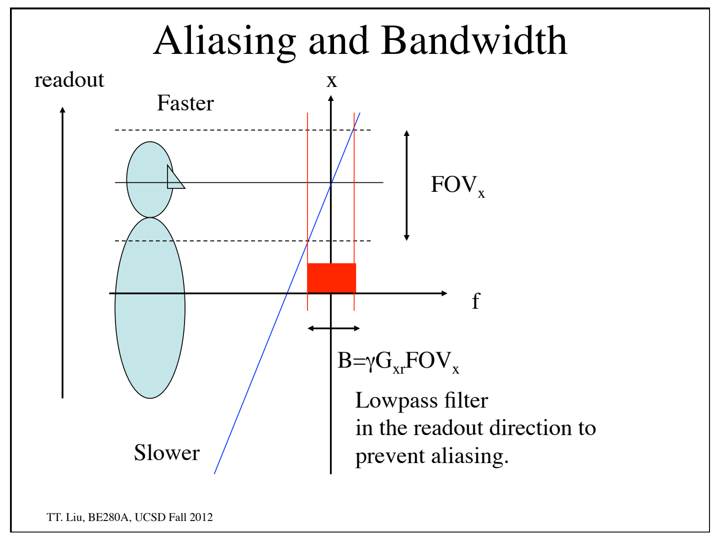

Aliasing and Bandwidth
readout
x
Faster
FOVx
f
B=γGxrFOVx
Lowpass filter
in the readout direction to
prevent aliasing.
Slower
TT. Liu, BE280A, UCSD Fall 2012

Figure 7-31 Default Axial Directions

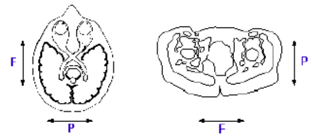
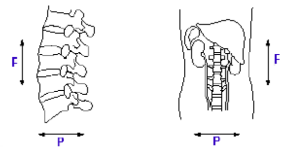


Figure 7-32 Default Sagittal and Coronal Directions



GE Medical Systems 2003