## HOMEWORK \#6

## Due at 5 pm on Friday 11/05/13

Homework policy: Homeworks can be turned in during class or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by $20 \%$ per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material.

Readings: Review sections 5.1-5.5 and 5.6.2. Read Section 5.7
For all problems, assume that $\frac{\gamma}{2 \pi}=4257 \mathrm{~Hz} / \mathrm{G}$

## Problem 1

Consider the object $m(x, y)=\operatorname{rect}(x+1, y+1)-\operatorname{rect}(x-1, y-1)$ that you looked at in Homework 3.
(a) Determine the values of $\Delta k_{x}$ and $\Delta k_{y}$ needed to avoid aliasing.
(b) Assume that the desired resolution in both the x and y directions is $\frac{1}{4} \mathrm{~cm}$. Determine $k_{x, \text { max }}$ and $k_{y, \text { max }}$ and also the number of voxels in the x and y directions. Assuming a spin-warp acquisition with one phase-encode line per repetition, what is the total scantime for TR = 100 ms ? (note: TR is the time between RF pulses).
(c) Assume that the readout time (i.e. time to cover one line in k-space) is much smaller than TR $=100 \mathrm{~ms}$. Develop and describe a simple solution that takes advantage of the geometry of the object to reduce total scan time by roughly a factor of 2 , while keeping the TR at 100 ms .

## Problem 2

Consider the 2D object: $f(x, y)=\cos (2 \pi x-2 \pi \sqrt{3} y) \operatorname{rect}(x / 4) \operatorname{rect}(\sqrt{3} y / 4)$
a) Sketch the object, labeling key features
b) Compute and sketch the Fourier Transform of the object, labeling key features.
c) Determine the values of $\Delta k_{x}$ and $\Delta k_{y}$ needed to avoid aliasing.
d) Assume that the desired resolution in the $x$ and $y$ directions is given by $1 / 2 \mathrm{~cm}$ and $\frac{1}{2 \sqrt{3}}$ cm . Determine $k_{x, \text { max }}$ and $k_{y, \max }$ and also the number of voxels in the x and y directions.
e) Draw and label gradient waveforms that will provide sufficient coverage of $k$-space, according to the specifications in parts c and d. Assume a spin-warp trajectory, a maximum available gradient of $6 \mathrm{G} / \mathrm{cm}$, and a sampling rate of 10 micro-seconds. Where possible, minimize the duration of the gradients (i.e. minimize the time of the initial readout dephaser necessary to get to the starting point in k-space).

## Problem 3

Consider the gradient waveforms shown in the figure. The full waveforms are shown in panels (a) and (b), and zoomed-in views are shown in (c) and (d). The analog-to-digital converter (ADC) is turned on during the flat parts of the readout (Gx) gradients with a sampling rate of $\Delta t$.
(a) Determine the sequence parameters (G1, G2 and G3, and $\Delta t$ ) to achieve the following image specifications: $\mathrm{FOV}_{\mathrm{x}}=192 \mathrm{~mm} ; \mathrm{FOV}_{\mathrm{y}}=192 \mathrm{~mm}, \delta_{x}=3 \mathrm{~mm}$ and $\delta_{y}=24 \mathrm{~mm}$.
(b) Draw the k -space trajectory (make sure to label units correctly).
(a) Gx gradient




