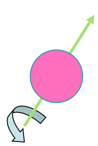
Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2013
MRI Lecture 1

Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

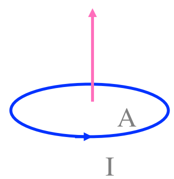
This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.

Nuclear Spin Rules

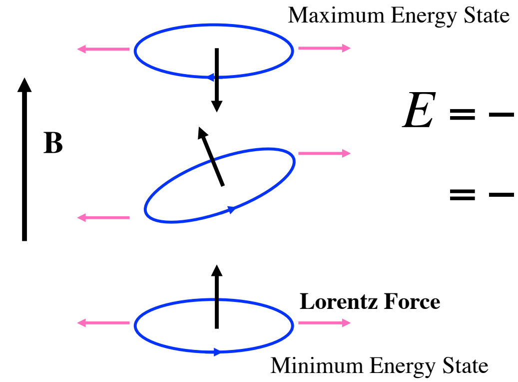
Number of Protons	Number of Neutrons	Spin	Examples
Even	Even	0	^{12}C, ^{16}O
Even	Odd	j/2	^{17}O
Odd	Even	j/2	^{1}H, ^{23}Na, ^{31}P
Odd	Odd	j	^{2}H

Classical Magnetic Moment



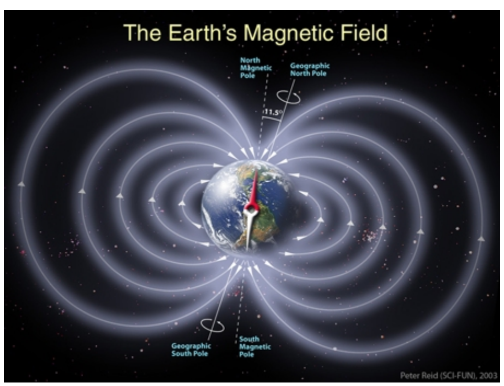
$$\vec{\mu} = IA\hat{n}$$

Energy in a Magnetic Field



$$E = -\vec{\mu} \bullet \mathbf{B}$$
$$= -\mu_z B$$

Energy in a Magnetic Field



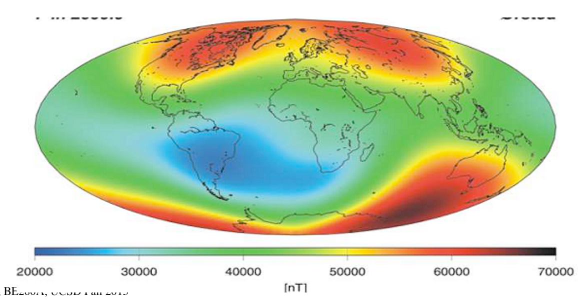
 www.qi-whiz.com/images/ Earth-magnetic-field.jpg

Magnetic Field Units

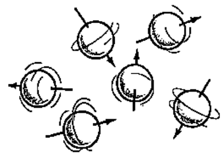
1 Tesla = 10,000 Gauss

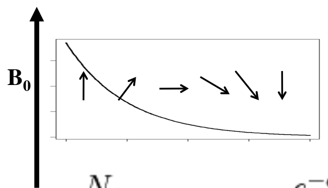
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

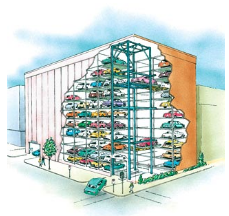


Boltzmann Distribution

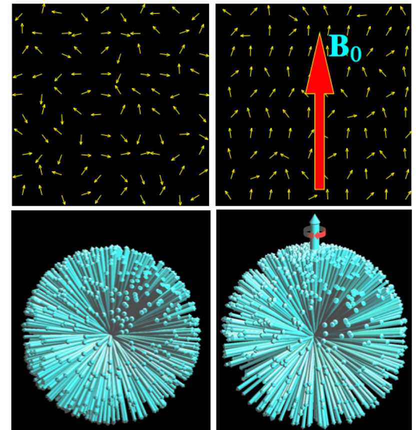




$$\frac{N_j}{N} \equiv \mathcal{P}(\epsilon_j) = \frac{e^{-\epsilon_j/(kT)}}{Q}.$$



Equilibrium Magnetization



$$\mathbf{M}_0 = N\langle \mu_z \rangle$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2\hbar^2 B/(4kT)$$

N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Hansen 2009

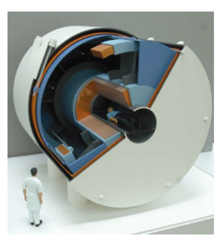
UCSD 3T

U. Minn. 7T

UIC 9.4T

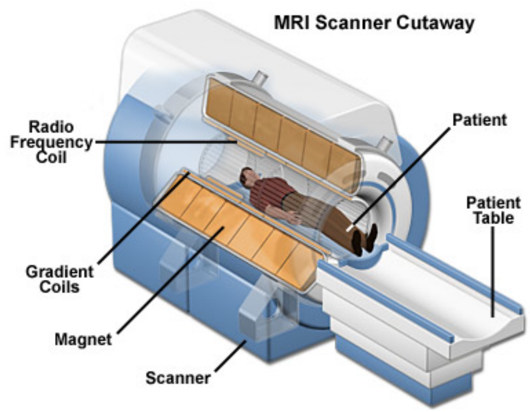
The 11.7 T MR magnet with members of the MRI team at Agilent's Magnet Technology Center in Yarnton, UK (from left to right): Darren Houlden, Andrew Winter, Paul Johnson, Nigel Haynes and Alistair Courtney.

11.7T NIH

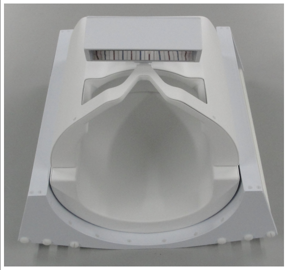


11.7T NeuroSpin France

MRI System



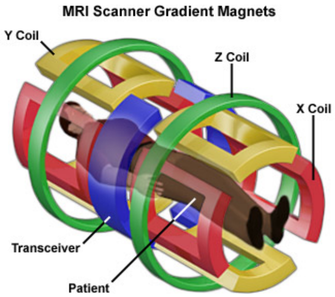
http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/fullarticle.html



Nova 32 channel

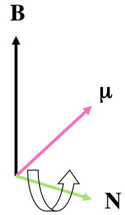
Siemens 32 channel

MRI Gradients



http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/fullarticle.html
http://gulfnews.com/business/economy/german-factories-race-to-meet-big-surge-in-orders-1.638066

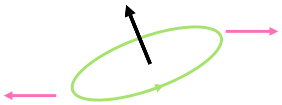
Torque



For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)

$\mathbf{N} = \mu \times \mathbf{B}$

Torque



Precession

Torque

$$\mathbf{N} = \mu \times \mathbf{B}$$

$$\frac{d\mathbf{S}}{dt} = \mathbf{N}$$

Change in Angular momentum

$$\frac{d\mathbf{S}}{dt} = \mu \times \mathbf{B}$$

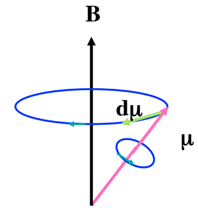
$$\mu = \gamma\,\mathbf{S}$$

Relation between magnetic moment and angular momentum

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

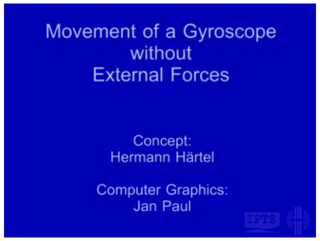


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma\,\mathbf{B}$$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertelmpg_e/gyros_free.mpg

Magnetization Vector

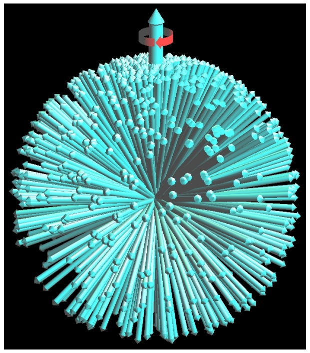
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

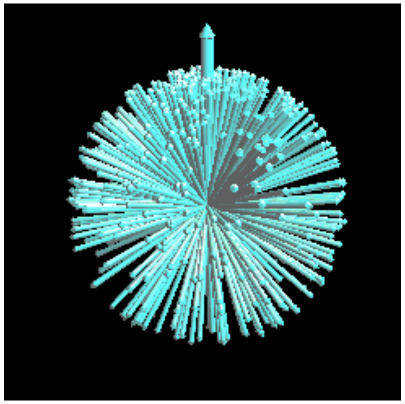
$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

$$\boxed{\frac{d\mathbf{M}}{dt} = \gamma\mathbf{M} \times \mathbf{B}}$$



Hansen 2009

RF Excitation



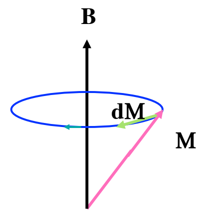
http://www.drcmr.dk/main/content/view/213/74/

Free precession about static field

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B}$$

$$= \gamma \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \gamma \begin{pmatrix} \hat{i}\left(B_z M_y - B_y M_z\right) \\ -\hat{j}\left(B_z M_x - B_x M_z\right) \\ \hat{k}\left(B_y M_x - B_x M_y\right) \end{pmatrix}$$



Free precession about static field

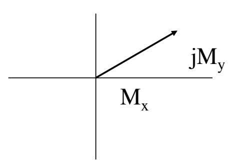
$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} B_z M_y - B_y M_z \\ B_x M_z - B_z M_x \\ B_y M_x - B_x M_y \end{bmatrix}$$

$$= \gamma \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Precession

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} 0 & B_0 & 0 \\ -B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Useful to define $M \equiv M_x + jM_y$



$$dM/dt = d/dt\left(M_x + iM_y\right)$$
$$= -j\gamma B_0 M$$

Solution is a time-varying phasor

$$M(t) = M(0)e^{-j\gamma B_0 t} = M(0)e^{-j\omega_0 t}$$

Question: which way does this rotate with time?

Gyromagnetic Ratios

Nucleus	Spin	Magnetic Moment	$\gamma/(2\pi)$ (MHz/Tesla)	Abundance
^{1}H	1/2	2.793	42.58	88 M
^{23}Na	3/2	2.216	11.27	80 mM
^{31}P	1/2	1.131	17.25	75 mM

Source: Haacke et al., p. 27

Larmor Frequency

$\omega = \gamma B$ Angular frequency in rad/sec

$f = \gamma B / (2\pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Notation and Units

1 Tesla = 10,000 Gauss

Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

$\gamma = 26752$ radians/second/Gauss

$\gamma = \gamma / 2\pi = 4258$ Hz/Gauss

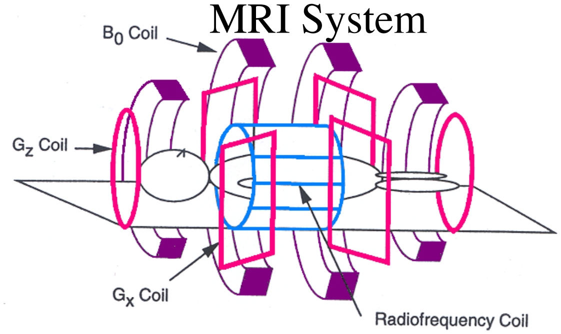
$= 42.58$ MHz/Tesla

Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z = B_0$, all the spins precess at the same frequency (ignoring chemical shift).

Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0 + \Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

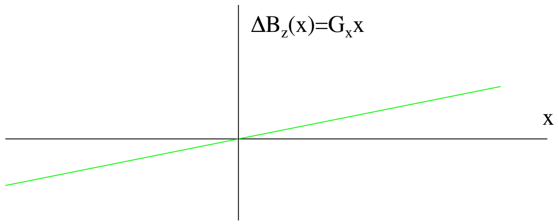
MRI System



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

Interpretation



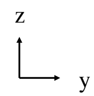
Spins Precess at γB_0- $\gamma G_x x$ (slower)

Spins Precess at γB_0

Spins Precess at at γB_0+ $\gamma G_x x$ (faster)

Gradient Fields

$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$
$$= B_0 + G_x x + G_y y + G_z z$$

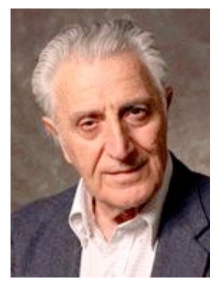


$$G_z = \frac{\partial B_z}{\partial z} > 0 \qquad G_y = \frac{\partial B_z}{\partial y} > 0$$

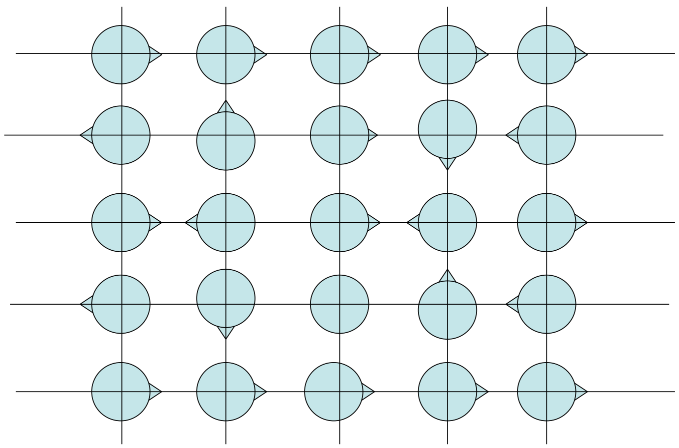
Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

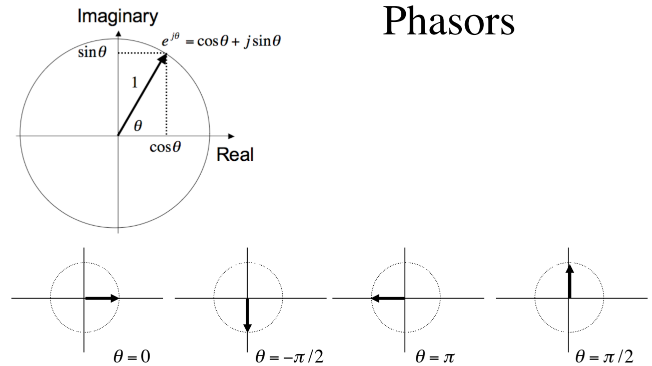
Spins



There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.
Erwin Hahn



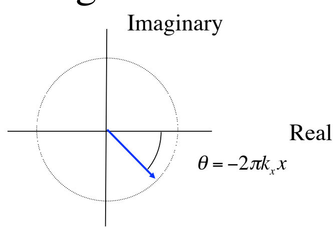
Phasors



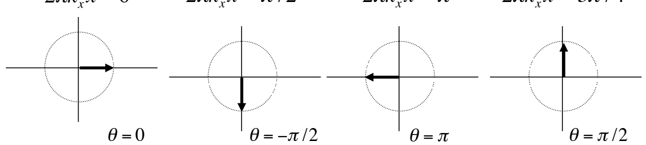
Phasor Diagram

$$G(k_x) = \int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

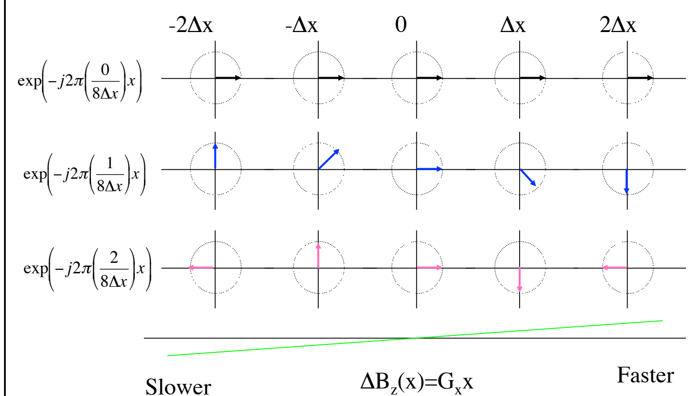
$\theta = -2\pi k_x x$

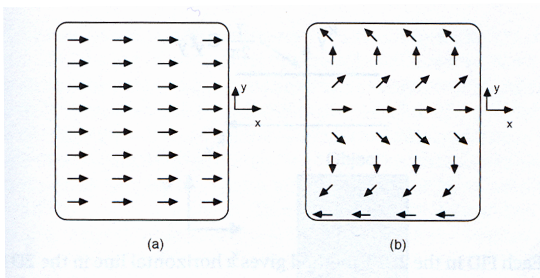


$k_x = 1;\ x = 0$	$x = 1/4$	$x = 1/2$	$x = 3/2$
$2\pi k_x x = 0$	$2\pi k_x x = \pi/2$	$2\pi k_x x = \pi$	$2\pi k_x x = 3\pi/4$



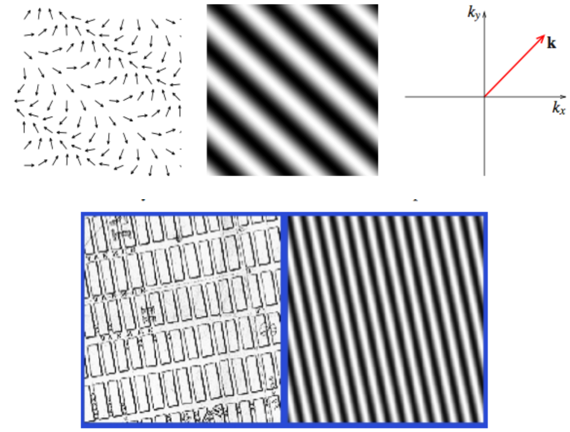
Interpretation



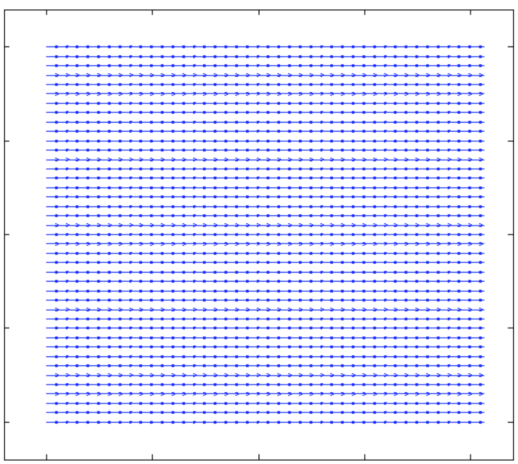


k_x=0; k_y=0 k_x=0; k_y≠0

Fig 3.12 from Nishimura

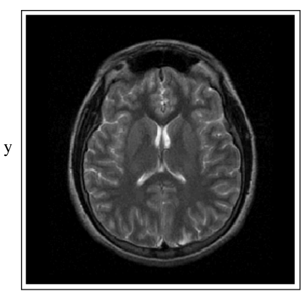


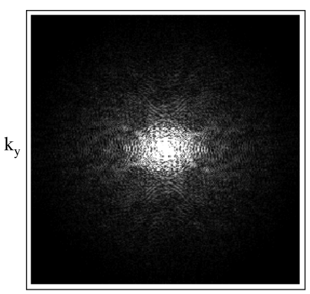
Hanson 2009



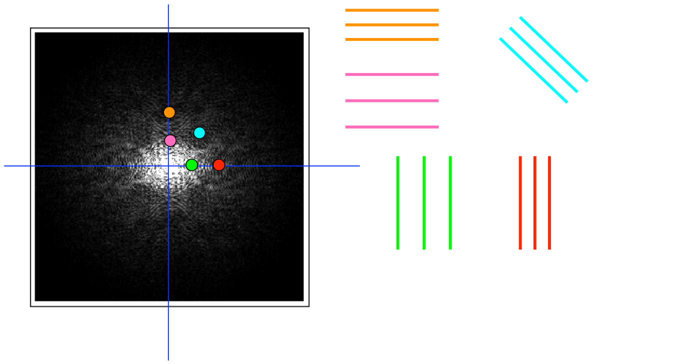
k-space

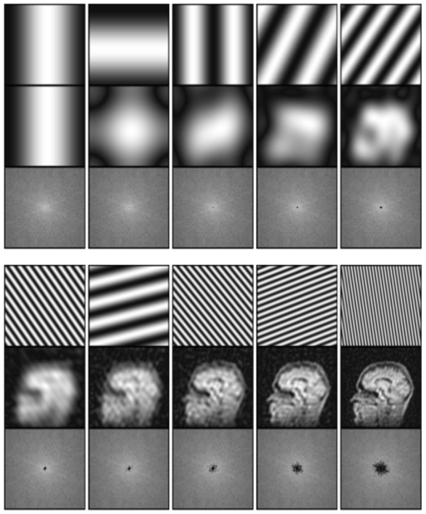
Image space k-space

y  x

k_y  k_x

↔ Fourier Transform





Hanson 2009

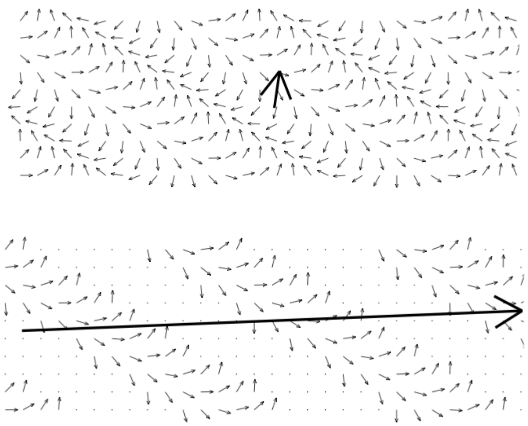
2D Fourier Transform

Fourier Transform

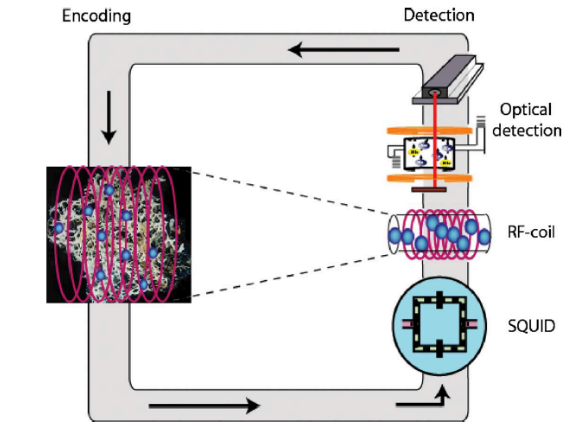
$$G(k_x,k_y) = F\left[g(x,y)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi\left(k_x x + k_y y\right)}dxdy$$

Inverse Fourier Transform

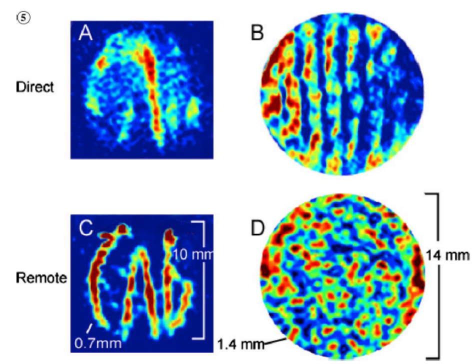
$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(k_x,k_y)e^{j2\pi\left(k_x x + k_y y\right)}dk_x dk_y$$



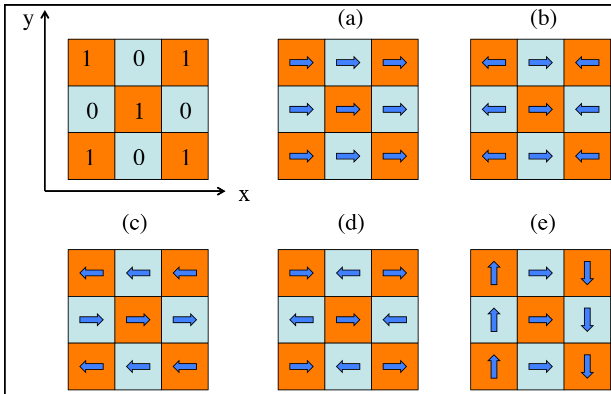
Hanson 2009



Seeley et al, JMR 2004



Seeley et al, JMR 2004



PollEv.com/be280a