Image Quality
Lecture 2

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Resident Physics Course
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Topics
Review MTF question
Noise
Receiver Operating Characteristics
Sampling and Aliasing
MTF = Fourier Transform (LTF)

**Figure 1:**

**Figure 2:**

10. Referring to Figure 1 which shows LSFs, and Figure 2 which shows the corresponding modulation transfer functions (MTFs), which MTF corresponds to LSF C?

A. MTF number 1  
B. MTF number 2  
C. MTF number 3

11. Referring to Figure 2 illustrating MTFs, the axes should be labeled _____ for the y-axis and _____ for the x-axis.

A. Relative amplitude, distance (mm)  
B. Spatial frequency (lp/mm), distance (mm)  
C. Lateral dimension (mm), Fresnel ratio  
D. Relative amplitude, spatial frequency (lp/mm)  
E. Relative amplitude, relative amplitude
What is Noise?

Fluctuations in either the imaging system or the object being imaged.

**Quantization Noise**: Due to conversion from analog waveform to digital number.

**Quantum Noise**: Random fluctuation in the number of photons emitted and recorded.

**Thermal Noise**: Random fluctuations present in all electronic systems. Also, sample noise in MRI

**Other types**: flicker, burst, avalanche - observed in semiconductor devices.

**Structured Noise**: physiological sources, interference
Histograms and Distributions

![Graph showing 3rd grade and 6th grade heights](image)

Bushberg et al 2001

Gaussian Distribution

![Gaussian distribution graph](image)

Bushberg et al 2001

1, 2, and 3 standard deviation intervals correspond to 68%, 95%, and 99% of the observations.
Poisson Process

Events occur at random instants of time at an average rate of \( \lambda \) events per second.

Examples: arrival of customers to an ATM, emission of photons from an x-ray source, lightning strikes in a thunderstorm.

\[ \lambda = \text{Average rate of events per second} \]
\[ \lambda t = \text{Average number of events at time } t \]
\[ \lambda t = \text{Variance in number of events} \]

Quantum Noise

For a Poisson process, the mean = variance, i.e. \( \bar{X} = \sigma^2 \)

Therefore, the standard deviation is given by \( \sigma = \sqrt{\bar{X}} \)

For X-ray systems, if the mean number of counts is \( N \), then the standard deviation in the number of counts is \( \sigma = \sqrt{N} \).

\[ \text{SNR} = \frac{N}{\sigma} = \frac{N}{\sqrt{N}}. \]
Poisson Distribution describes x-ray counting statistics.
Gaussian distribution is good approximation to Poisson when $\sigma = \sqrt{X}$
G79. A series of measurements has a mean of 100 counts. A range of $\pm \sigma$ is ______.

A. 95–105  
B. 90–100  
C. 68–137  
D. 50–150  
E. 33–167

G80. To achieve a standard deviation of 2%, ______ counts must be collected.

A. 400  
B. 1,414  
C. 2,500  
D. 10,000  
E. 40,000

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G79. B The standard deviation $\sigma$ is the square root of the mean, in this case $\sqrt{100} = 10$. There is a 68% probability that any random reading will fall within $\sigma$ of the mean, and a 95% probability that it will fall within $2\sigma$ of the mean.

G80. C The percent standard deviation, $%\sigma = (\sigma / \bar{N}) \times 100 = (\sqrt{N} / \bar{N}) \times 100 = 100 / \sqrt{N}$. In this case $100 / \sqrt{N} = 2$, so $N = 2500$. 
G73. A radioactive sample is counted many times, and the mean is 2500 counts. 96% of the readings will lie between ____ and ____ counts.
A. 2300 2500
B. 2400 2500
C. 2400 2600
D. 2450 2550
E. 2500 2700

G73. C If a large number of measurements are made, approximately 67% will fall between ±σ, and 96% between ±2σ of the mean. The standard deviation σ = \sqrt{\text{N}}, or 50 in this case. 2500 ± (2 × σ) = 2400 – 2600.

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?
A. 1000
B. 3162
C. 10,000
D. 40,000
E. 100,000

D70. D A 95% confidence interval means the counts must fall within two standard deviations (SD) of the mean (N). Error limit = 1% = 2 SD/N, but SD = N^{1/2}.

Thus 0.01 = 2(N^{1/2})/N = 2/N^{1/2}.
[0.01]^2 = 4/N
N = 40,000.
Contrast Resolution

Lower row shows effect of *structure noise*.

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**THE 2 × 2 DECISION MATRIX**

<table>
<thead>
<tr>
<th></th>
<th>Actually Abnormal</th>
<th>Actually Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnosed as Abnormal</td>
<td>True Positive (TP)</td>
<td>False Positive (FP)</td>
</tr>
<tr>
<td>Diagnosed as Normal</td>
<td>False Negative (FN)</td>
<td>True Negative (TN)</td>
</tr>
</tbody>
</table>
Sensitivity = \( \frac{TP}{TP + FN} \)
= Fraction of people who have the disease who test positive

Specificity = \( \frac{TN}{TN + FP} \)
= Fraction of people who do not have the disease who test negative

Positive Predictive Value = \( \frac{TP}{TP + FP} \)
= Probability patient is actually abnormal when diagnosed as abnormal

Negative Predictive Value = \( \frac{TN}{TN + FN} \)
= Probability patient is actually normal when diagnosed as normal.

True Positive Fraction = \( \frac{TP}{TP + FN} \)
= Sensitivity
= Probability of Detection

False Positive Fraction = \( \frac{FP}{FP + TN} \)
= 1 - Specificity
= Probability of False Alarm

Receiver operating characteristic (ROC) curve plots True Positive Fraction vs. False Positive Fraction
Area is a measure of detectability
In Figure 6, showing an ROC curve, the X axis should be labeled (circle all that are correct):
A. True Positive Fraction
B. False Positive Fraction
C. Sensitivity
D. Specificity
E. 1 - Specificity

In Figure 6 showing the ROC curves, the Y axis should be labeled (circle all that are correct):
A. True Positive Fraction
B. False Positive Fraction
C. Sensitivity
D. Specificity
E. 1 - Specificity

Of the ROC curves in Figure 6:
28. Curve number represents pure guessing.
29. Curve number represents the best diagnostic approach.
30. Curve number represents an AUC value of about 0.7.
Nyquist Frequency \( = F_N = \frac{1}{2\Delta} \), Sampling Pitch

If \( f > F_N \), then aliasing will occur

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Sampling in Image Space
Sampling in k-space

Image Quality, T.T. Liu, Spring 2006
Smoothing of Projections in CT

Projection

Beam Width

Smoothed Projection

\[ W = \frac{2}{\Delta s} \]
\[ \delta = 1/W = \Delta s/2 \]