Bioengineering 208
Magnetic Resonance Imaging

Winter 2007
Lecture 1

Topics

• Review of NMR basics
• Hardware Overview
• Quadrature Detection
Boltzmann Distribution

\[ E = \mu_z B_0 \]
\[ \Delta E = \gamma h B_0 \]
\[ E = -\mu_z B_0 \]

Number of Spins Down
Number of Spins Up = \( e^{-\Delta E/kT} \)

Ratio = 0.999990 at 1.5T !!!
Corresponds to an excess of about 10 up spins per million

Magnetization Vector

Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

\[ \mathbf{M} = \frac{1}{V} \sum_{\text{protons in } V} \mathbf{u}_i \]

\[ \frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} \]

http://www.easymeasure.co.uk/principlesmri.aspx
RF Excitation

At equilibrium, net magnetization is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

B₁ radiofrequency field tuned to Larmor frequency and applied in transverse (xy) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the z-axis.

- lab frame of reference

http://www.eecs.umich.edu/~tnoll/BME516/
Slide Credit: T.T. Liu

Bloch Equation

\[
\frac{dM}{dt} = M \times \gamma B - \frac{M_x i + M_y j}{T_2} - \frac{(M_z - M_0)k}{T_1}
\]

Precession  
Transverse Relaxation  
Longitudinal Relaxation

\[ \omega_0 = \gamma B \]

i, j, k are unit vectors in the x, y, z directions.
Relaxation: Z-component

\[ M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1} \]

Relaxation: Transverse Component

\[
M \equiv M_x + jM_y
\]

\[
dM/dt = d/dt(M_x + iM_y) = -j(\omega_0 + 1/T_2)M
\]

\[
M(t) = M(0)e^{-j\omega_0 t}e^{-t/T_2} \quad \omega_0 = \gamma B
\]

Relaxation: Summary

1) Longitudinal component recovers exponentially.

2) Transverse component precesses and decays exponentially.
**Gradient Echo**

![Diagram of Gradient Echo](image)

**Small Tip Angle Example**

\[
B_1(t) = B_1 \text{rect}\left(\frac{t - \frac{\tau}{2}}{\tau}\right)
\]

\[
M_z(t, z) = jM_0 \exp(-j\omega(z)\tau) \int_0^\tau \exp(j\omega(z)s)\omega_1 \text{rect}\left(\frac{s - \frac{\tau}{2}}{\tau}\right) ds
\]

\[
= jM_0 \exp(-j\omega(z)\tau/2)F_{1D}\left(\omega_1 \text{rect}\left(\frac{t}{\tau}\right)\right)_{\tau=(\gamma/2\pi)G_z} z
\]

\[
= jM_0 \exp(-j\omega(z)\tau/2)\omega_1 \tau \sin(\gamma G_z^2 \frac{\tau}{2\pi} z)
\]

\[
\Delta z = \frac{1}{\frac{\gamma}{2\pi} G_z \tau}
\]

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Hardware Overview

Three fields:
• Main Field ($B_0$)
  • Polarize Spins
• Gradient Fields ($G_{[XYZ]}$)
  • Map space into frequency
    $\frac{\partial B_z}{\partial [XYZ]}$
• RF Fields ($B_1$)
  • Change the latitude or zenith angle of (‘tips’) spins

How do we decide on $B_0$?
$\Delta E = \gamma h B_0$
$M_0 \propto \Delta E$

∴ Bigger is better!

3T Human @UCSD  7T Rodent @UCSD  7T Human @U.Minn.  9.4T Human @UIC
Main Field \((B_0)\)

Energy in a Magnetic Field:

\[
E = \frac{1}{2\mu_0} \int B^2 dV
\]

For \(B=3T\) over \(1m^3\):

\[
E = \frac{1}{2(1.25 \times 10^{-6})} \cdot 9 = 3.6 MJ
\]

= dropping a 1000kg car from 360m high

Heat of Vaporization of He = 2.5 KJ/l

During a quench, \(R\) goes from 0 to \(\sim 100\Omega\), \(I\sim 100A\), so \(P=I^2R\sim 1MW\)

A quench can boil off \(3.6MJ/2.5KJ/l=1400l\) of Helium

in \(3.6MJ/1MW \sim 3.6s \) !!!!

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Main Field \((B_0)\)

Wavelength \((\lambda)\) of RF:

In Vacuum:

- 5m @ 60MHz (1.5T)
- 1m @ 300MHz (7T)

In Brain:

- 12cm @ 300MHz (7T)\(^1\)

When \(\lambda\) is not large compared to object, standing waves form. This is referred to as Dielectric Resonance. RF inhomogeneity during receive is fixed by scaling, but RF transmit inhomogeneity is much more difficult to address.

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\(^1\) Vaughan et al, MRM 46 p24 2001
Gradient Fields

\[ G_x \equiv \frac{\partial B_z}{\partial X} \quad G_y \equiv \frac{\partial B_z}{\partial Y} \quad G_z \equiv \frac{\partial B_z}{\partial Z} \]

How big do gradient fields need to be?
• Shortest soft tissue \( T_2^* \sim 1\text{ms} \)
• For 0.2mm resolution in 1ms:
  \[ G = \frac{K_{\text{max}}}{\gamma T} = \frac{(0.5/0.2\text{mm})}{(4257\text{Hz}/G)(1\text{ms})} = 5\text{G/cm} \]
• To fill \( 1\text{m}^3 \) with 5G/cm gradients in 0.2ms requires:
  \[ P = \frac{E}{T} = \frac{1/2\mu_0 \int B^2 dV}{T} = \frac{1/2\mu_0 (B_{\text{RMS}}(5G/cm))^2 (1\text{m}^3)}{0.2\text{ms}} = 500\text{KW} \]
  About 3 simultaneous Rolling Stones concerts
• Modern gradient systems are also up against dB/dt limits for peripheral nerve stimulation (~50T/s)
• For diffusion or ultrashort \( T_2^* \) imaging, more G would help a lot

RF Fields

How big do RF fields need to be?
• Shortest soft tissue \( T_2^* \sim 1\text{ms} \)
• To flip spins by 90° (0.25 rotations) in 0.2ms:
  \[ B_1 = \frac{0.25}{\gamma T} = \frac{0.25}{(4257\text{Hz}/G)(0.2\text{ms})} = 0.24G \]
• RF power absorption by the body is a complex function of frequency, conductivity, and geometry, but at 0.24G, approximately 200W/Kg are deposited in human tissue at 3T. Thus, for a 100Kg person, the RF system must supply 20KW of deposited power, or about 40KW of total power, assuming 50% losses to the coil, cabling, reflections, and radiation.
Quadrature Reception

Original quadrature detection: separate coils and A/D for I and Q

Because $\omega_0 >>$ bandwidth, 2 coils are not needed for phase detection. However, second coil does increase SNR.
Quadrature Reception

Digital quadrature requires 2x faster sampling, but eliminates I/Q imbalance, and what happens to a DC offset in the A/D?

\[ V_0 \cos(\omega_0 t) + N \]
\[ V_0 \sin(\omega_0 t) + N \]

\[ \cos(\omega_0 + \text{IF}) t \]
\[ \sin(\text{IF} t) \]

\[ \cos(\omega_0 + \text{IF}) t = \{1, 0, -1, 0, \ldots\} \]
\[ \sin(\text{IF} t) = \{0, 1, 0, -1, \ldots\} \]

Summary:
1. Quadrature RF coil is NOT needed to detect MR signal phase.
2. Quadrature RF coil improves SNR by \( \sqrt{2} \)
3. Digital quadrature detection:
   1. Eliminates I/Q imbalance
   2. Moves DC offset in ADC to edge of image
   3. Requires 2x higher sampling rate than separate I/Q