Excitation k-space

Pauly et al 1989
Moving Spins

So far we have assumed that the spins are not moving (aside from thermal motion giving rise to relaxation), and contrast has been based upon $T_1$, $T_2$, and proton density. We were able to achieve different contrasts by adjusting the appropriate pulse sequence parameters.

Biological samples are filled with moving spins, and we can also use MRI to image the movement. Examples: blood flow, diffusion of water in the white matter tracts. In addition, we can also sometimes induce motion into the object to image its mechanical properties, e.g. imaging of stress and strain with MR elastography.

Phase of Moving Spin

\[ \Delta B_z(x) \]

\[ \Delta B_x(x) \]

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Phase of a Moving Spin

\[ \varphi(t) = -\int_0^t \Delta \omega(\tau) d\tau \]
\[ = -\int_0^t \gamma \Delta B(\tau) d\tau \]
\[ = -\int_0^t \gamma \tilde{G}(\tau) \cdot \tilde{r}(\tau) d\tau \]
\[ = -\gamma \int_0^t [G_x(\tau)x(\tau) + G_y(\tau)y(\tau) + G_z(\tau)z(\tau)] d\tau \]

Phase of Moving Spin

Consider motion along the x-axis

\[ x(t) = x_0 + vt + \frac{1}{2}at^2 \]

\[ \varphi(t) = -\gamma \int_0^t G_x(\tau)x(\tau) d\tau \]
\[ = -\gamma \int_0^t G_x(\tau) \left[ x_0 + vt + \frac{1}{2}a\tau^2 \right] d\tau \]
\[ = -\gamma \left[ x_0 \int_0^t G_x(\tau) d\tau + v \int_0^t G_x(\tau) \tau d\tau + \frac{a}{2} \int_0^t G_x(\tau) \tau^2 d\tau \right] \]
\[ = -\gamma \left[ x_0 M_0 + vM_1 + \frac{a}{2} M_2 \right] \]
Phase of Moving Spin

\[ \varphi(t) = -\gamma \left[ x_0 M_0 + v M_1 + \frac{a}{2} M_2 \right] \]

\[ M_0 = \int_0^T G_x(\tau) d\tau \quad \text{Zeroth order moment} \]
\[ M_1 = \int_0^T G_x(\tau) \tau d\tau \quad \text{First order moment} \]
\[ M_2 = \int_0^T G_x(\tau) \tau^2 d\tau \quad \text{Second order moment} \]

Flow Moment Example

\[ M_0 = \int_0^T G_x(\tau) d\tau = 0 \]
\[ M_1 = \int_0^T G_x(\tau) \tau d\tau = -\int_0^T G_0 \tau d\tau + \int_T^{2T} G_0 \tau d\tau \]
\[ = G_0 \left[ -\frac{T^2}{2} + \frac{T^2}{2} \right] \]
\[ = G_0 \left[ -\frac{T^2}{2} + \frac{4T^2}{2} - \frac{T^2}{2} \right] = G_0 T^2 \]
Phase Contrast Angiography (PCA)

\[ \varphi_1 = -\gamma v_x M_1 = \gamma v_x G_0 T^2 \]

\[ \varphi_2 = -\gamma v_x M_1 = -\gamma v_x G_0 T^2 \]

\[ \Delta \varphi = \varphi_1 - \varphi_2 = 2\gamma v_x G_0 T^2 \]

\[ v_x = \frac{\Delta \varphi}{2G_0 T^2} \]

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PCA example

White = Flow direction AP (\(\downarrow\))
White = Flow direction RL (\(\rightarrow\))


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### Aliasing in PCA

Define VENC as the velocity at which the phase is 180 degrees.

\[ VENC = \frac{\pi}{\gamma G_0 T^2} \]

Because of phase wrapping, the velocity of spins flowing faster than VENC is ambiguous.

### Aliasing Solutions

Use data from regions with slower flow to resolve aliasing.

Use multiple VENC values so that the phase differences are smaller than π radians.

\[ \phi_1 = \pi \frac{v_x}{VENC_1} \]
\[ \phi_2 = \pi \frac{v_x}{VENC_2} \]
\[ \phi_1 - \phi_2 = \pi v_x \left( \frac{1}{VENC_1} - \frac{1}{VENC_2} \right) \]
Velocity k-space

A bipolar gradient introduces a phase modulation across velocities of the form \( q(v_x) = -\gamma v_x G_0 T^2 \).

We can make measurements with different amounts of phase modulation and then integrate over velocities to obtain

\[
M(k_{vx}) = \int_{-\infty}^{\infty} m(v_x) e^{j q(v_x)} dv_x
\]

\[
= \int_{-\infty}^{\infty} m(v_x) e^{-j \gamma v_x G_0 T^2} dv_x
\]

\[
= \int_{-\infty}^{\infty} m(v_x) e^{-j 2 \pi k_{vx} v_x} dv_x
\]

\[
= F[m(v_x)] \text{ with } k_{vx} = \frac{\gamma}{2\pi} G_0 T^2
\]

By making measurements with bipolar gradients of varying amplitudes/durations and taking the inverse transform of the measurements, we can obtain the velocity distribution.

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Velocity k-space

\[
M(k_x, k_{vx}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x, v_x) e^{-j 2 \pi k_x x} e^{-j 2 \pi k_{vx} v_x} dx dv_x
\]

In addition, we can apply imaging gradients so that we can eventually obtain the velocity distribution at each point in space. A full k-space acquisition would then yield 6 dimensions -- 3 spatial dimensions and 3 velocity dimensions.

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During readout moving spins within the object will accumulate phase that is in addition to the phase used for imaging. This leads to

1) Net phase at echo time $TE = 2T$.

2) An apparent shift in position of the object.

3) Blurring of the object due to a quadratic phase term.

### Flow Artifacts

**Plug Flow**

All moving spins in the voxel experience the same phase shift at echo time.

**Laminar Flow**

Spins have different phase shifts at echo time. The dephasing causes the cancelation and signal dropout.
Flow Compensation

At TE both the first and second order moments are zero, so both stationary and moving spins have zero net phase.

Inflow Effect

Prior to imaging

Relaxed spins flowing in

Saturated spins

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Cerebral Blood Flow (CBF)

CBF = Perfusion

= Rate of delivery of arterial blood to a capillary bed in tissue.

Units: \( \frac{\text{ml of Blood}}{100 \text{ grams of tissue} \times \text{minute}} \)

Typical value is 60 ml/(100g-min) or 60 ml/(100 ml-min) = 0.01 s\(^{-1}\), assuming average density of brain equals 1 gm/ml
High CBF

Low CBF

Time

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Arterial Spin Labeling

- Magnetically tag inflowing arterial blood
- Wait for tagged blood to flow into imaging slice
- Acquire image of tissue + tagged blood
- Apply control pulse that doesn’t tag blood
- Acquire control image of tissue
- Control image-tag image = blood image

Arterial Spin Labeling (ASL)

1: Tag by Magnetic Inversion → Wait → Acquire image

2: Control → Wait → Acquire image

Control - Tag \( \propto \) CBF

Credit: Wen-Ming Luh
**Arterial Spin Labeling (ASL)**

- Water protons as freely diffusible tracers

![Diagram of imaging slice and alternative inversion](image)

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**Multislice CASL and PICORE**

- **CASL**
- **PICORE**
- **QUIPSS II**

![Images of brain scans](image)

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*Courtesy of Wen-Ming Luh*  
*Credit: E. Wong*
Diffusion

2D random walk

N random steps of length d

\[ <\Delta x^2> = Nd^2 = 2DT \]

D = diffusivity

In brain:

D \approx 0.001 \text{ mm}^2/\text{s}

For T=100 msec,

\[ \Delta x \approx 15 \mu \]

Diffusion Weighting

Assume \( \delta \ll T \)

\[ \varphi(t_1) \approx -\gamma G_0 x(t_1) \delta \]

\[ \varphi(t_2) \approx +\gamma G_0 x(t_2) \delta \]

Net Phase

\[ \varphi = \varphi(t_1) + \varphi(t_2) = \gamma G_0 \left[ x(t_2) - x(t_1) \right] \delta = \gamma G_0 \Delta x \delta \]

Average Squared Phase

\[ \langle \varphi^2 \rangle = \gamma^2 G_0^2 \delta^2 \langle (\Delta x)^2 \rangle = \gamma^2 G_0^2 \delta^2 2DT \]

Signal

\[ S \propto e^{\langle \varphi^2 \rangle/2} = e^{-\gamma^2 G_0^2 \delta^2 DT} = e^{-bD} \]

where \( b = \gamma^2 G_0^2 \delta^2 T \)

A more careful analysis yields \( b = \gamma^2 G_0^2 \delta^2 (T - \delta / 3) \)
After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

Restricted Diffusion

D depends on direction

Diffusion tensor:
- 3 values of $D$
- 3 angles

Credit: Larry Frank
Diffusion Imaging Example

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Q-ball imaging

Tuch et al, Neuron 2003

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Fiber Tract Mapping

Mori et al., MRM 2002

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fMRI

MRI studies brain anatomy. Functional MRI (fMRI) studies brain function.

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http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm
**fMRI Setup**

![fMRI Setup Diagram]

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http://defiant.ssc.uwo.ca/3dty_web/fmri4dummies.htm

**fMRI Acquisition**

- **High spatial resolution**
  - **MP-RAGE**
    - Voxel volume: 1 mm$^3$
    - Imaging time: 6 min

- **High temporal resolution**
  - **EPI**
    - Voxel volume: 45 mm$^3$
    - Imaging time: 60 msec

Buxton 2002
Visual Activation

Flickering Checkerboard
OFF (60 s) - ON (60 s) - OFF (60 s) - ON (60 s) - OFF (60 s)

Brain Activity

Time

Source: Kwong et al., 1992

Finger Tapping Task

Source: "Defiant", 2005

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http://defiant.ssc.uwo.ca/body_web/fmri4dummies.htm
Oxygen binds to the iron atoms to form oxyhemoglobin $\text{HbO}_2$
Release of $\text{O}_2$ to tissue results in deoxyhemoglobin $d\text{HBO}_2$

Effect of $d\text{HBO}_2$

$d\text{HBO}_2$ is paramagnetic due to the iron atoms. As it becomes oxygenated, it becomes less paramagnetic.

d$\text{HBO}_2$ perturbs the local magnetic fields. As blood becomes more deoxygenated, the amount of perturbation increases and there is more dephasing of the spins. Thus as $d\text{HBO}_2$ increases we find that $T_2^*$ decreases and the amplitude $\exp(-TE/T_2^*)$ image of a $T_2^*$ weighted image will decrease. Conversely as $d\text{HBO}_2$ decreases, $T_2^*$ increases and we expect the signal amplitude to go up.
BOLD Effect

Blood Oxygen Level Dependent signal

- neural activity $\rightarrow$ ↑ blood flow $\rightarrow$ ↑ oxyhemoglobin $\rightarrow$ ↑ T2* $\rightarrow$ ↑ MR signal

Basal state
- normal flow
- basal level [Hb]
- basal CBV
- normal MRI signal

Activated state
- increased flow
- decreased [Hb] (lower field gradients around vessels)
- increased CBV
- increased MRI signal (from lower field gradients)

Source: Jovicich

Source: fMRIB Brief Introduction to fMRI

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm
BOLD Dynamics

Fig. 17.5
Rat Forepaw Stimulation
(Mandeville, et al, 1999)
BOLD and Vascular Dynamics

Cardiac Imaging

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Cardiac Tagging

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Cardiac Diffusion and Strain

Dou et al, MRM 2003

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