Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
X-Rays/CT Lecture 2
View Aliasing

CT System Generations

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Suetens 2002
CT System

(a) 

(b) 

(c) 

(d) 

Fan Beam

\[ \theta = \alpha + \beta \]

\[ r = R \sin \alpha \]
**Fan Beam**

\[ \theta = \alpha + \beta \]

\[ r = R \sin \alpha \]

**Spiral CT**

From http://www.sprawls.org/resources/CTIMG/classroom.htm

Nearest Neighbor Interpolation

Linear Interpolation
Longitudinal Aliasing in Spiral CT

Multislice CT
Poisson Process

Events occur at random instants of time at an average rate of $\lambda$ events per second.

Examples: arrival of customers to an ATM, emission of photons from an x-ray source, lightning strikes in a thunderstorm.

Assumptions:

1) Probability of more than 1 event in an small time interval is small.

2) Probability of event occurring in a given small time interval is independent of another event occurring in other small time intervals.

\[ P[N(t) = k] = \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \]

$\lambda$ = Average rate of events per second

$\lambda t$ = Average number of events at time $t$

$\lambda t$ = Variance in number of events

Probability of interarrival times

\[ P[T > t] = e^{-\lambda t} \]
Example

A service center receives an average of 15 inquiries per minute. Find the probability that 3 inquiries arrive in the first 10 seconds.

\[ \lambda = \frac{15}{60} = 0.25 \]
\[ \lambda t = 0.25(10) = 2.5 \]

\[ P[N(t = 10) = 3] = \frac{(2.5)^3}{3!} \exp(-2.5) = .2138 \]

Quantum Noise

Fluctuation in the number of photons emitted by the x-ray source and recorded by the detector.

\[ P_k = \frac{N_0^k \exp(-N_0)}{k!} \]

\( P_k \) : Probability of emitting k photons in a given time interval.

\( N_0 \) : Average number of photons emitted in that time interval = \( \lambda t \)
Transmitted Photons

\[ Q_k = \frac{(pN_0)^k \exp(-pN_0)}{k!} \]

- \( Q_k \): Probability of \( k \) photons making it through object
- \( N_0 \): Average number of photons emitted in that time interval = \( \lambda t \)
- \( p = \exp(-\int \mu dz) \) = probability of proton being transmitted

Example

Over the diagnostic energy range, the photon density is approximately \( 2.5 \times 10^{10} \) photons/cm\(^2\) / \( R \) where \( R \) stands for roentgen (unit for X-ray exposure).

A typical chest x-ray has an exposure of 50 mR. For transmission in regions devoid of bone,

\[ p = \exp(-\int \mu dz) = 0.05 \]

What are the mean and standard deviation of the number of photons that make it it to a 1 mm\(^2\) detector?

\[ pN_0 = 0.05 \cdot 2.5 \times 10^{10} \cdot 0.05 \cdot 0.1 = 6.25 \times 10^5 \text{ photons} \]

mean = \( 6.25 \times 10^5 \) photons

standard deviation = \( \sqrt{6.25 \times 10^5} = 790 \) photons
Contrast and SNR for X-Rays

Contrast = \( C = \frac{\Delta I}{I} \)

\[
\text{SNR} = \frac{\Delta I}{\sigma_I} = \frac{\text{Mean difference in # of photons}}{\text{Standard Deviation of # photons}}
\]

\[
= \frac{CpN_0}{\sqrt{pN_0}} = C\sqrt{pN_0}
\]

\[
C = \frac{\Delta I}{I} = \frac{N_0\left(\exp(-\mu_1 L) - \exp\left(-\left(\mu_1 (L - W) + \mu_2 W\right)\right)\right)}{N_0 \exp(-\mu_1 L)}
\]

\[
\text{SNR} = \frac{CN_0 A \exp(-\mu_1 L)}{\sqrt{N_0 A \exp(-\mu_1 L)}} = C\sqrt{N_0 A \exp(-\mu_1 L)}
\]
Signal to Noise Ratio for CT

$$SNR = \frac{C\bar{\mu}}{\sigma_\mu}$$

$$\approx 0.4kC\mu d^{3/2}/\sqrt{MN/T}$$

$C =$ contrast
$\bar{\mu} =$ mean attenuation
$N =$ mean number of transmitted photon
$T =$ spacing between detectors
$M =$ number of views
$\rho_0 =$ bandwidth of Ram-Lak filter $\approx k/d$ where $d =$ width of detector
$k =$ scaling constant, order unity

CT Applications
Virtual Colonoscopy