Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2005
X-Rays/CT Lecture 2

View Aliasing
Poisson Process

Events occur at random instants of time at an average rate of \( \lambda \) events per second.

Examples: arrival of customers to an ATM, emission of photons from an x-ray source, lightning strikes in a thunderstorm.

Assumptions:
1) Probability of more than 1 event in a small time interval is small.
2) Probability of event occurring in a given small time interval is independent of another event occurring in other small time intervals.

Poisson Process

\[
P[N(t) = k] = \frac{(\lambda t)^k}{k!} \exp(-\lambda t)
\]

\( \lambda \) = Average rate of events per second
\( \lambda t \) = Average number of events at time \( t \)
\( \lambda t \) = Variance in number of events

Probability of interarrival times

\[
P[T > t] = e^{-\lambda t}
\]
Example

A service center receives an average of 15 inquiries per minute. Find the probability that 3 inquiries arrive in the first 10 seconds.

\[ \lambda = \frac{15}{60} = 0.25 \]
\[ \lambda t = 0.25(10) = 2.5 \]

\[ P[N(t=10) = 3] = \frac{(2.5)^3}{3!} \exp(-2.5) = .2138 \]

Quantum Noise

Fluctuation in the number of photons emitted by the x-ray source and recorded by the detector.

\[ P_k = \frac{N_0^k \exp(-N_0)}{k!} \]

\[ P_k \]: Probability of emitting \( k \) photons in a given time interval.

\[ N_0 \]: Average number of photons emitted in that time interval = \( \lambda t \)

Transmitted Photons

\[ Q_k = \frac{\left(pN_0\right)^k \exp(-pN_0)}{k!} \]

\[ Q_k \]: Probability of \( k \) photons making it through object

\[ N_0 \]: Average number of photons emitted in that time interval = \( \lambda t \)

\[ p = \exp(-\int \mu dz) \] = probability of proton being transmitted
Example

Over the diagnostic energy range, the photon density is approximately $2.5 \times 10^{10}$ photons/cm$^2$/R where R stands for roentgen (unit for X-ray exposure).

A typical chest x-ray has an exposure of 50 mR. For transmission in regions devoid of bone,

$$p = \exp(-\mu L) = 0.05$$

What are the mean and standard deviation of the number of photons that make it to a 1 mm$^2$ detector?

$$pN_0 = 0.05 \cdot 2.5 \times 10^{10} \cdot 0.05 \cdot (1)^2 = 6.25 \times 10^7$$ photons

Mean = $6.25 \times 10^7$ photons

Standard deviation = $\sqrt{6.25 \times 10^7} = 790$ photons

## Contrast and SNR for X-Rays

Contrast = $C = \frac{\Delta I}{I}$

$$SNR = \frac{\Delta I}{\sigma_I}$$

Mean difference in # of photons

Standard Deviation of # photons

$$= \frac{CpN_0}{\sqrt{pN_0}}$$

$$= C \sqrt{pN_0}$$
Signal to Noise Ratio for CT

$$SNR = \frac{C\mu}{\sigma\mu}$$

$$= \frac{C\pi}{\frac{2\mu\sigma}{3}}$$

$$= 0.44\left(\frac{d}{\rho_0}\right)^{1/3}\sqrt{SNR/T}$$

$C$ = contrast
$\mu$ = mean attenuation
$N$ = mean number of transmitted photon
$T$ = spacing between detectors
$M$ = number of views
$\rho_0$ = bandwidth of Ram-Lak filter = $kd$ where $d$ = width of detector
$k$ = scaling constant, order unity

CT Applications

Virtual Colonoscopy