Small Tip Angle Approximation

\[ M_r(t,z) = jM_0 \exp(-j\omega(z)t) \int_0^t \exp(j\omega(z)s)\omega_t(s)ds \]

For symmetric pulse of length \( \tau \)

\[ M_r(\tau,z) = jM_0 \exp(-j\omega(z)\tau/2) \int_{-\tau/2}^{\tau/2} \exp(j2\tau f(z)s)\omega_t(s + \tau/2)ds \]

\[ = jM_0 \exp(-j\omega(z)\tau/2)F \{ \omega_t(t + \tau/2) \} \left| _{f(z)=-\frac{\tau}{2\pi}G_z} \right. \]
Cardiac Tagging

Excitation k-space

Pauly et al 1989
Excitation k-space

Static Inhomogeneities

In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called “shim” coils, and the process of making the field more uniform is called “shimming”. In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.
**Static Inhomogeneities**

The spatial nonuniformity in the field can be modeled by adding an additional term to our signal equation.

\[ s_r(t) = \int_V M(\vec{r}, t) dV = \int_x \int_y \int_z M(x, y, z, 0) e^{-i/T_2^*(\vec{r})} e^{-j\omega_0 t} e^{-j\omega_0 \vec{r} \cdot \vec{r}} \exp \left( -j \gamma \int_0^t \vec{G}(\tau) \cdot \vec{r} d\tau \right) dx dy dz \]

The effect of this nonuniformity is to cause the spins to dephase with time and thus for the signal to decrease more rapidly. To first order this can be modeled as an additional decay term of the form

\[ s_r(t) = \int_x \int_y \int_z M(x, y, z, 0) e^{-i/T_2^*(\vec{r})} e^{-i/T_2^*(\vec{0})} e^{-j\omega_0 t} \exp \left( -j \gamma \int_0^t \vec{G}(\tau) \cdot \vec{r} d\tau \right) dx dy dz \]

**T_2^* decay**

The overall decay has the form.

\[ \exp \left( -t / T_2^* (\vec{r}) \right) \]

where

\[ \frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} \]

Due to random motions of spins. Not reversible.  
Due to static inhomogeneities. Reversible with a spin-echo sequence.
**Spin Echo**

Discovered by Erwin Hahn in 1950.

The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be $T_2$ dephasing due to random motion of spins.

*There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.* Erwin Hahn

Image: Larry Frank
Phase at time $\tau$

$$\varphi(\tau) = \int_0^\tau -\omega_E(\vec{r}) dt = -\omega_E(\vec{r}) \tau$$

Phase after 180 pulse

$$\varphi(\tau^+) = \omega_E(\vec{r}) \tau$$

Phase at time $2\tau$

$$\varphi(2\tau) = -\omega_E(\vec{r}) \tau + \omega_E(\vec{r}) \tau = 0$$
Different tissues exhibit different relaxation rates, $T_1$, $T_2$, and $T_2^*$. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).
**Saturation Recovery Sequence**

Gradient Echo

\[ I(x, y) = \rho(x, y) \left[ 1 - e^{-\frac{TR}{T_1(x,y)}} \right] e^{-\frac{TE}{T_2^*(x,y)}} \]

Spin Echo

\[ I(x, y) = \rho(x, y) \left[ 1 - e^{-\frac{TR}{T_1(x,y)}} \right] e^{-\frac{TE}{T_2(x,y)}} \]

**T1-Weighted Scans**

Make TE very short compared to either \( T_2 \) or \( T_2^* \). The resultant image has both proton and \( T_1 \) weighting.

\[ I(x, y) \approx \rho(x, y) \left[ 1 - e^{-\frac{TR}{T_1(x,y)}} \right] \]
T2-Weighted Scans

Make TR very long compared to $T_1$ and use a spin-echo pulse sequence. The resultant image has both proton and $T_2$ weighting.

$$I(x, y) = \rho(x, y)e^{-\frac{TE}{T_2}}$$

Proton Density Weighted Scans

Make TR very long compared to $T_1$ and use a very short TE. The resultant image is proton density weighted.

$$I(x, y) \approx \rho(x, y)$$
Example

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FLASH sequence

\[ I(x,y) = \rho(x,y) \left[\frac{1 - e^{-\frac{TR}{T_1(x,y)}}}{1 - e^{-\frac{TR}{T_2(x,y)}}}\right] \sin \theta \exp \left(\frac{-TE}{T_2^*}\right) \]

Signal intensity is maximized at the Ernst Angle

\[ \theta_E = \cos^{-1} \left( \exp \left(\frac{-TR}{T_1}\right) \right) \]

FLASH equation assumes no coherence from shot to shot. In practice this is achieved with RF spoiling.

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FLASH sequence

\[ \theta_e = \cos^{-1}(\exp(-TR/T_1)) \]

Inversion Recovery

\[ I(x,y) = \rho(x,y) \left[ 1 - 2e^{-TI(x,y)} + e^{-TR/T_1(x,y)} \right] e^{-TE/T_2(x,y)} \]

Intensity is zero when inversion time is

\[ TI = -T_1 \ln \left[ \frac{1 + \exp(-TR/T_1)}{2} \right] \]
Echoplanar Imaging

Thomas Liu, BE280A, UCSD, Fall 2006
GE Medical Systems 2003