Moving Spins

So far we have assumed that the spins are not moving (aside from thermal motion giving rise to relaxation), and contrast has been based upon $T_1$, $T_2$, and proton density. We were able to achieve different contrasts by adjusting the appropriate pulse sequence parameters.

Biological samples are filled with moving spins, and we can also use MRI to image the movement. Examples: blood flow, diffusion of water in the white matter tracts. In addition, we can also sometimes induce motion into the object to image its mechanical properties, e.g. imaging of stress and strain with MR elastography.
Phase of a Moving Spin

\[
\varphi(t) = -\int_0^t \Delta \omega(\tau) d\tau \\
= -\int_0^t \gamma \Delta B(\tau) d\tau \\
= -\int_0^t \gamma \tilde{G}(\tau) \cdot \tilde{r}(\tau) d\tau \\
= -\gamma \int_0^t \left[ G_x(\tau)x(\tau) + G_y(\tau)y(\tau) + G_z(\tau)z(\tau) \right] d\tau
\]
Phase of Moving Spin

Consider motion along the x-axis

\[ x(t) = x_0 + vt + \frac{1}{2}at^2 \]

\[ \varphi(t) = -\gamma \int_0^t G_x(\tau)x(\tau)d\tau \]

\[ = -\gamma \int_0^t G_x(\tau) \left[ x_0 + vt + \frac{1}{2}a\tau^2 \right] d\tau \]

\[ = -\gamma \left[ x_0 \int_0^t G_x(\tau)d\tau + v \int_0^t G_x(\tau)\tau d\tau + \frac{a}{2} \int_0^t G_x(\tau)\tau^2 d\tau \right] \]

\[ = -\gamma \left[ x_0 M_0 + v M_1 + \frac{a}{2} M_2 \right] \]

Zeroth order moment

First order moment

Second order moment

\[ M_0 = \int_0^t G_x(\tau)d\tau \]

\[ M_1 = \int_0^t G_x(\tau)\tau d\tau \]

\[ M_2 = \int_0^t G_x(\tau)\tau^2 d\tau \]
**Flow Moment Example**

\[ M_0 = \int_0^T G_x(\tau) d\tau = 0 \]
\[ M_1 = \int_0^T G_x(\tau) d\tau \]
\[ = -\int_0^T G_0 d\tau + \int_0^{2T} G_0 d\tau \]
\[ = G_0 \left[ -\frac{T^2}{2} + \frac{T^2}{2} \right] \]
\[ = G_0 \left[ -\frac{T^2}{2} + \frac{4T^2}{2} - \frac{T^2}{2} \right] = G_0 T^2 \]

**Phase Contrast Angiography (PCA)**

\[ \varphi_1 = -\gamma \nu_x M_1 = \gamma \nu_x G_0 T^2 \]
\[ \varphi_2 = -\gamma \nu_x M_1 = -\gamma \nu_x G_0 T^2 \]
\[ \Delta \varphi = \varphi_1 - \varphi_2 = 2\gamma \nu_x G_0 T^2 \]
\[ \nu_x = \frac{\Delta \varphi}{2G_0 T^2} \]
PCA example

- $G_0$

Define $VENC$ as the velocity at which the phase is 180 degrees.

$$VENC = \frac{\pi}{\gamma G_0 T^2}$$

Because of phase wrapping, the velocity of spins flowing faster than $VENC$ is ambiguous.
Aliasing Solutions

- Use data from regions with slower flow to avoid aliasing.
- Use multiple VENC values so that the phase differences are smaller than $\pi$ radians.

$$q_1 = \pi \frac{v_x}{\text{VENC}_1}$$

$$q_2 = \pi \frac{v_x}{\text{VENC}_2}$$

$$q_1 - q_2 = \pi v_x \left( \frac{1}{\text{VENC}_1} - \frac{1}{\text{VENC}_2} \right)$$

Velocity k-space

A bipolar gradient introduces a phase modulation across velocities of the form $-\gamma v_{x} G_{0} T^2$

We can make measurements with different amounts of phase modulation and then integrate over velocities to obtain

$$M(k_{v_x}) = \int_{-\infty}^{\infty} m(v_x) e^{i \gamma (v_x) \cdot k_{v_x}} dv_x$$

$$= \int_{-\infty}^{\infty} m(v_x) e^{-j v_x G_0 T^2} dv_x$$

$$= \int_{-\infty}^{\infty} m(v_x) e^{-j 2 \pi k_{v_x} v_x} dv_x$$

$$= F[m(v_x)] \text{ with } k_{v_x} = \frac{\gamma}{2\pi} G_0 T^2$$

By making measurements with bipolar gradients of varying amplitudes/durations and taking the inverse transform of the measurements, we can obtain the velocity distribution.
Velocity k-space

\[ M(k_x, k_{vx}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x, v_x) e^{-j2\pi k_x x} e^{-j2\pi k_{vx} v_x} dx dv_x \]

In addition, we can apply imaging gradients so that we can eventually obtain the velocity distribution at each point in space. A full k-space acquisition would then yield 6 dimensions -- 3 spatial dimensions and 3 velocity dimensions.

Flow Artifacts

During readout moving spins within the object will accumulate phase that is in addition to the phase used for imaging. This leads to

1) Net phase at echo time TE = 2T.
2) An apparent shift in position of the object.
3) Blurring of the object due to a quadratic phase term.
Flow Artifacts

Plug Flow

All moving spins in the voxel experience the same phase shift at echo time.

Laminar Flow

Spins have different phase shifts at echo time. The dephasing causes the cancelation and signal dropout.

Flow Compensation

At TE both the first and second order moments are zero, so both stationary and moving spins have zero net phase.
Inflow Effect

Prior to imaging

Relaxed spins flowing in

Saturated spins

Time of Flight Angiography

Thomas Liu, BE280A, UCSD, Fall 2006
Cerebral Blood Flow (CBF)

CBF = Perfusion
= Rate of delivery of arterial blood to a capillary bed in tissue.

Units: \[
\frac{\text{(ml of Blood)}}{\text{(100 grams of tissue)(minute)}}
\]

Typical value is 60 ml/(100g-min) or 60 ml/(100 ml-min) = 0.01 s\(^{-1}\), assuming average density of brain equals 1 gm/ml
Arterial Spin Labeling

- Magnetically tag inflowing arterial blood
- Wait for tagged blood to flow into imaging slice
- Acquire image of tissue + tagged blood
- Apply control pulse that doesn’t tag blood
- Acquire control image of tissue
- Control image-tag image = blood image
Arterial Spin Labeling (ASL)

1: Tag by Magnetic Inversion
   Wait
   Acquire image

2: Control
   Wait
   Acquire image

Control - Tag $\propto$ CBF

Credit: Wen-Ming Luh

Arterial Spin Labeling (ASL)

- water protons as freely diffusible tracers

imaging slice
alternative inversion

Mz(blood)

control

$\Delta M$

tag

t

Courtesy of Wen-Ming Luh

Thomas Liu, BE280A, UCSD, Fall 2006
Diffusion

\[ <\Delta x^2> = N d^2 = 2DT \]
\[ D = \text{diffusivity} \]

In brain:
\[ D = 0.001 \text{ mm}^2/\text{s} \]
For \( T = 100 \) msec,
\[ \Delta x \approx 15 \mu \]

Credit: Larry Frank
Assume $\delta << T$

\[ \varphi(t_1) \approx -\gamma G_0 x(t_1) \delta \]

\[ \varphi(t_2) \approx +\gamma G_0 x(t_2) \delta \]

Net Phase

\[ \varphi = \varphi(t_1) + \varphi(t_2) = \gamma G_0 \left[ x(t_2) - x(t_1) \right] \delta = \gamma G_0 \Delta x \delta \]

Average Squared Phase

\[ \langle \varphi^2 \rangle = \gamma^2 G_0^2 \delta^2 \left( \Delta x \right)^2 = \gamma^2 G_0^2 \delta^2 2DT \]

Signal

\[ S \propto e^{-(\varphi^2)/2} = e^{-\gamma^2 G_0^2 \delta^2 2DT} = e^{-bD} \quad \text{where} \quad b = \gamma^2 G_0^2 \delta^2 T \]

A more careful analysis yields $b = \gamma^2 G_0^2 \delta^2 (T - \delta/3)$

After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.
**Restricted Diffusion**

D depends on direction

![Diagram of diffusion tensor with 3 values and 3 angles](image)

**Diffusion Imaging Example**

![Diffusion imaging example](image)

Thomas Liu, BE280A, UCSD, Fall 2006

Credit: Larry Frank
Q-ball imaging

Tuch et al, Neuron 2003

Fiber Tract Mapping

Mori et al., MRM 2002
fMRI

MRI studies brain anatomy.

Functional MRI (fMRI) studies brain function.

fMRI Setup

Thomas Liu, BE280A, UCSD, Fall 2006

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm
fMRI Acquisition

High spatial resolution

High temporal resolution

MP-RAGE
Voxel volume: 1 mm$^3$
Imaging time: 6 min

EPI
Voxel volume: 45 mm$^3$
Imaging time: 60 msec

Visual Activation

Flickering Checkerboard
OFF (60 s) - ON (60 s) - OFF (60 s) - ON (60 s) - OFF (60 s)

Brain Activity

Source: Kwong et al., 1992

http://defiant.ssc.uwo.ca/3ody_web/fmri4dummies.htm
Finger Tapping Task

Hemoglobin

**A Molecule To Breathe With**

Hemoglobin

Oxygen binds to the iron atoms to form oxyhemoglobin HbO₂
Release of O₂ to tissue results in deoxyhemoglobin dHBO₂

http://www.people.virginia.edu/~rjh9u/hemoglob.html
Effect of dHBO$_2$

dHBO$_2$ is paramagnetic due to the iron atoms. As it becomes oxygenated, it becomes less paramagnetic.

dHBO$_2$ perturbs the local magnetic fields. As blood becomes more deoxygenated, the amount of perturbation increases and there is more dephasing of the spins. Thus as dHBO$_2$ increases we find that $T_2^*$ decreases and the amplitude $\exp(-T\over T_2^*)$ image of a $T_2^*$ weighted image will decrease. Conversely as dHBO$_2$ decreases, $T_2^*$ increases and we expect the signal amplitude to go up.
BOLD Effect
Blood Oxygen Level Dependent signal

↑ neural activity ➔ ↑ blood flow ➔ ↑ oxyhemoglobin ➔ ↑ T2* ➔ ↑ MR signal

Source: [fMRIB Brief Introduction to fMRI](http://www.fmrib.ox.ac.uk)

BOLD Dynamics

Stimulus ➔ ATP ➔ ADP ➔ Neural Activity ➔ CMRGlc ➔ CBV ➔ BOLD Signal

Source: [fMRI for Dummies](http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm)

Thomas Liu, BE280A, UCSD, Fall 2006

Credit: Rick Buxton
BOLD Dynamics

Fig. 17.5  Rat Forepaw Stimulation
(Mandeville, et al, 1999)

BOLD and Vascular Dynamics

Thomas Liu, BE280A, UCSD, Fall 2006
Timeline

Michael Crichton, 1999

“Most people”, Gordon said, “don’t realize that the ordinary hospital MRI works by changing the quantum state of atoms in your body ... But the ordinary MRI does this with a very powerful magnetic field - say 1.5 tesla, about twenty-five thousand times as strong as the earth’s magnetic field. We don’t need that. We use Superconducting QUantum Interference Devices, or SQUIDs, that are so sensitive they can measure resonance just from the earth’s magnetic field. We don’t have any magnets in there”.

Thomas Liu, BE280A, UCSD, Fall 2006

J. Clarke, UC Berkeley
J. Clarke, UC Berkeley

\[ B_0 = 132 \mu T \]
\[ G = 100 \mu T/m \]
48 projections
1.5 min. acquisition

Seeley et al, JMR 2004

Thomas Liu, BE280A, UCSD, Fall 2006
Seeley et al, JMR 2004