Bioengineering 280A
Principles of Biomedical Imaging

Fall Quarter 2013
MRI Lecture 4

Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field $B_0$, gradient fields (two of three shown), and radiofrequency field $B_1$. Image, caption: copyright Nishimura, Fig. 3.15

RF Excitation

From Levitt, Spin Dynamics, 2001

http://www.drcmr.dk/main/content/view/213/74/
RF Excitation

At equilibrium, net magnetization is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

$B_1$, radiofrequency field tuned to Larmor frequency and applied in transverse (xy) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the z-axis.

- lab frame of reference

On-Resonance Excitation

Hanson 2009

http://www.drcmr.dk/JavaCompass/

Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.
a) Laboratory frame behavior of $\mathbf{M}$

b) Rotating frame behavior of $\mathbf{M}$

http://www.eecs.umich.edu/~dnol

http://www.mrinstruments.com/

$\mathbf{B}_1(t) = 2B_1(t)\cos(\omega t)\mathbf{i}$

$= B_1(t)(\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}) + B_1(t)(\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j})$
Precession

Analogous to motion of a gyroscope

$\frac{d\mu}{dt} = \mu \times \gamma B$

Precesses at an angular frequency of

$\omega = \gamma B$

This is known as the Larmor frequency.

Rotating Frame Bloch Equation

$\frac{dM_{\text{rot}}}{dt} = M_{\text{rot}} \times \gamma B_{\text{eff}}$

$B_{\text{eff}} = B_{\text{rot}} + \frac{\omega_{\text{rot}}}{\gamma}$, $\omega_{\text{rot}} = \begin{bmatrix} 0 \\ 0 \\ -\omega \end{bmatrix}$

Note: we use the RF frequency to define the rotating frame. If this RF frequency is on-resonance, then the main $B_0$ field doesn’t cause any precession in the rotating frame. However, if the RF frequency is off-resonance, then there will be a net precession in the rotating frame that is given by the difference between the RF frequency and the local Larmor frequency.

Let $B_{\text{rot}} = B_1(t)\hat{i} + B_0\hat{k}$

$B_{\text{eff}} = B_{\text{rot}} + \frac{\omega_{\text{rot}}}{\gamma}$

$= B_1(t)\hat{i} + \left(B_0 - \frac{\omega}{\gamma}\right)\hat{k}$

If $\omega = \omega_0$

$= \gamma B_0$

Then $B_{\text{eff}} = B_1(t)\hat{i}$

Flip angle

$\theta = \int_0^t \omega(t) \, dt$

where

$\omega(t) = \gamma B_1(t)$
Example
\[ \tau = 400 \ \mu \text{sec}; \ \theta = \pi/2 \]
\[ B_i = \frac{\theta}{\gamma \tau} = \frac{\pi/2}{2\pi (4257 \text{Hz} / G) (400 e - 6)} = 0.1468 \ G \]
**Small Tip Angle Approximation**

For small $\theta$

$M_z = M_0 \cos \theta \approx M_0$

$M_{xy} = M_0 \sin \theta \approx M_0 \theta$

**Excitation k-space**

$k(\tau, t) = \frac{\gamma}{2\pi} \int \tau G_z(s) ds$

$M, \theta \exp(-j2\pi k(\tau, t)z)$

$2M, \theta \exp(-j2\pi k(\tau, t)z)$

$M, \theta$
Excitation k-space

At each time increment of width $\Delta t$, the excitation $B(t)$ produces an increment in magnetization of the form

$$\Delta M_x = jM_0\gamma B(t)\Delta t$$

(small tip angle approximation)

In the presence of a gradient, this will accumulate phase of the form

$$\varphi = \gamma \int_0^t G_z(s)\,ds$$

such that the incremental magnetization at time $t$ is

$$\Delta M_x(t, z; \tau) = jM_0\gamma B(t)\exp\left(-j\gamma \int_0^t G_z(s)\,ds\right)\Delta t$$

Integrating over all time increments, we obtain

$$M_x(t, z) = jM_0\gamma \int_0^t B(t)\exp\left(-j\gamma \int_0^t G_z(s)\,ds\right)\,dt$$

where

$$k(t, \tau) = \frac{\gamma}{2\pi} \int_0^t G_z(s)\,ds$$

Refocusing

$$M_x(t, z) = jM_0\gamma \int_0^t B(t)\exp\left(-j2\pi k(t, \tau)z\right)\,dt$$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B(t)$ at the k-space point $k(t, \tau)$.

Slice Selection

$$M_x(t, z) = jM_0\gamma \int_0^t B(t)\exp\left(-j2\pi k(t, \tau)z\right)\,dt$$

This has the form of a Fourier transform, where we are integrating the contributions of the field $B(t)$ at the k-space point $k(t, \tau)$.
Gradient Echo

- RF
- $G_x(t)$: Slice select gradient
- $G_y(t)$: Slice refocusing gradient
- $G_z(t)$: Spins all in phase at $k_{zx0}$

ADC

Slice Selection

- Slice
- $z$-slice
- $f$: $\text{rect}(\tau)$
- $\Delta f = \frac{1}{\tau} = \frac{\gamma G \Delta z}{2\pi}$
- $\text{sinc}(t/\tau)$

Example

$M_x(s) = M_0 \cos(4\pi s)$

$F(M_x(s)) = \frac{M_0}{2} (\delta(k_x-2) + \delta(k_x+2))$

$\gamma = 4 \text{ G/cm}$

$\frac{\gamma}{2\pi} \gamma = 4 \text{ cm}^{-1}$; $T = 235 \mu\text{sec}$

with small tip angle approximation $\theta \approx \frac{\pi}{2}$

Compare with $\sin \left( \frac{\theta}{6} \right) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} = 0.5236$

Question: Should we use $\theta = \frac{\pi}{4}$ instead?

Exercise: Sketch the quiver diagrams corresponding to the contributions of the two RF pulses and show that they produce the desired pattern.
Multi-dimensional Excitation k-space

\[ M_{\omega}(t,r) = jM_0 \int_{-\infty}^{\infty} \omega_0(\tau) \exp\left(-j\gamma \int_0^\tau G(s) \cdot r ds\right) d\tau \]

\[ = jM_0 \int_{-\infty}^{\infty} \omega_0(\tau) \exp\left(j2\pi k(\tau) \cdot r\right) d\tau \]

where \[ k(\tau) = -\frac{\gamma}{2\pi} \int_0^\tau G(t') dt' \]

Pauly et al 1989

Excitation k-space

Excitation k-space

Cardiac Tagging

Panych MRM 1999