BENG280A, Principles of Biomedical Imaging 10/23/15
Fall Quarter 2014

HOMEWORK #4
Due at 5 pm on Friday 10/30/15

Homework policy: Homeworks can be turned in during class prior to the due date or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material. It is recommended that you make a copy of the homework for yourself (e.g. scan it in) before you turn it in.

Required Readings: Review the notes from class and prior readings as needed.

Problems:

1. Consider the CT k-space filter $G(k) = |k|w(k)$ where $w(k)$ is a windowing function. For each of the following window functions, use MATLAB to plot the k-space filter. Also, derive analytical expressions for the inverse Fourier transforms of the k-space filter *G(k)* when using the following window functions:

   a) The Ram-Lak Filter with $w(k) = rect\left(\frac{k}{2k_{\max}}\right)$.

   b) A Hanning window defined as $w(k) = rect\left(\frac{k}{2k_{\max}}\right)\left(0.5 + 0.5\cos\left(\frac{\pi k}{k_{\max}}\right)\right)$.

   c) Use MATLAB to plot out and compare the inverse transforms you derived in parts (a) and (b). Comment on the relative advantages and disadvantages of the two filters for CT reconstruction.

   NOTE: For the MATLAB plots, you may assume the $k_{\max} = 1$. Choose your sampling intervals in both the Fourier and spatial domains such that the plots adequately demonstrate the features of the function.

2. In this exercise, you will perform a filtered backprojection of the square object you looked at in HW3. Note that you will want to try to reuse the MATLAB code that you used to compute your projections and backprojection for HW3. Before proceeding, make sure your projections and MATLAB code from last week gave a good result for the backprojection (i.e. should look similar to the backprojection movie shown in class and posted on the class website). Make sure to include the reconstructed images in the work you turn in.

   a) Assume that the projections are sampled with a spatial resolution of 0.5 mm and the FOV is 40 mm. Make sure your projections from HW3 look okay with these parameters.
   b) Choose $k_{\max} = 1\ \text{mm}^{-1}$. Use the MATLAB conv function to filter your projections with the Ram-Lak filter expression you found in problem 1a. Note you will want to keep the "central" portion of the output of the conv function as your filtered projection (see the 'same' option in conv). Backproject the filtered projections and compare the reconstructed image to what you got by just backprojecting.
   c) Now filter with the Hanning Windowed filter. How does this change the reconstructed image? Provide an explanation for why the images look different.
   d) Next consider doing the filtering in the Fourier domain. Use the MATLAB fft function to compute the Fourier transforms (1D) of your projections. Multiply by the Ram-Lak filter response and then you use ifft to compute the inverse transform. Compare the real part of your

answer to the filtered responses you obtained in part (b). If they look the same, then you can proceed with doing the filtered back projections.

3. Consider the object $f(x,y) = 2rect(x/2)rect(y/2)\cos(4\pi x + 2\pi y)$:
   a. Sketch the object, making sure to label key features.
   b. Derive and sketch the Fourier transform of the object, making sure to label key features
   c. Derive and sketch the projections at 0 degrees and 90 degrees. Justify your answer.
   d. Determine the angle that has the maximum projection. Provide an explicit expression and sketch the projection at this angle.

Matlab Exercise 1: In this exercise you will use MATLAB to plot out the complex function $g(x) = \exp(j2\pi k_x x)$.
Enter in the following MATLAB commands (you will want to save this in a script, so you can easily modify):

```
j = sqrt(-1);
x = -10:.1:10;
kx = .1;
g = exp(j*kx*2*pi*x);
figure(1);clf;
quiver3(x,zeros(size(x)),zeros(size(x)),zeros(size(x)),real(g),imag(g));
```

1) Describe the main features of the plot and explain the main features – i.e. what is the period of repetition and the "handedness of the helix". You will find it useful to use the "rotate" button on the MATLAB plotting interface to look at the helix from different views.
2) Which way do the arrows point at x = 0, x = 2.5 and x = 5.0? Explain how the directions match up with the analytical expressions of the function.
3) What happens when you make `kx = -0.1`? Explain what is going on.
4) What happens when you make `kx = 0.2`? Explain what you observe and compare this to the original plot (e.g. from part 1).

Next enter in the following commands:

```
span = -10:.5:10;
[x,y] = meshgrid(span,span);
kx = .1;ky = .1;
g = exp(j*2*pi*(kx*x + ky*y));
figure(2);clf;
quiver(x,y,real(g),imag(g));
grid;
```

1) Explain what you are seeing. What function are we plotting? How does the way in which we are plotting the function differ from the way we did it in the first half of this exercise?
2) What is the fundamental period of the object – does this match the analytical prediction?
3) Which way do the arrows point at (0,0), (5,5), and (2.5, 2.5)? Do these match the analytical prediction?
4) How does the plot change when you make kx = -0.1? Explain the differences.
5) How does the plot change when you make kx = 0.05? Explain the differences.
6) Try your own combination of values of kx and ky (different from what was done above) and explain the main features of what you are seeing.