BENG280A, Principles of Biomedical Imaging 11/30/15
Fall Quarter 2015

HOMEWORK #7
Due at 5 pm on Wednesday 12/2/15

Homework policy: Homeworks can be turned in during class prior to the due date or to the TA's mailbox in the Graduate Student Lounge. Late homeworks will be marked down by 20% per day. If you know that you need to turn in a homework late because of an emergency or academic travel, please let the TA know ahead of time. Collaboration is encouraged on homework assignments, however, the homework that you submit should reflect your own understanding of the material. It is recommended that you make a copy of the homework for yourself (e.g. scan it in) before you turn it in.

Required Readings: Read Chapter 6 and Sections 7.1 through 7.4.

For all problems, assume that $\frac{\gamma}{2\pi} = 4257$ Hz/G

Problem 1
Do problem 4.3 from Nishimura (in other words, show that Eqn 4.15 is a solution to Eqn 4.14) ; In addition, use MATLAB to plot out the solution for initial conditions of (a) $M_z(0) = 0$; (b) $M_z(0) = -M_0/2$; and (c) $M_z(0) = -M_0$. Assume a T_1 of 1.5 seconds. For each of the initial conditions, determine the time at which the magnetization recovers to $0.95M_0$. Is this time the same for all initial conditions? – explain your finding.

Problem 2
Problem 7.3 in Nishimura.

Problem 3
You are asked to design an RF and gradient pulse sequence that achieves the following profile: $M_{xy}(z) = \frac{1}{4}\left(1+\cos(4\pi z)\right)^2$. The maximum available B_1 radiofrequency field is 0.25 Gauss and the maximum available gradient is 5 G/cm.

a) First compute the Fourier transform of the pulse profile. You may find it useful to use the convolution/multiplication theorem.
b) Next use the small tip angle approximation to determine the desired flip angles for each pulse. Compute the temporal width of each pulse required to achieve the desired flip angles, assuming that each pulse uses the maximum available B_1 field. (Although each pulse has a finite width, you may assume for the rest of the problem that each pulse acts as if it were a Dirac delta function positioned at the center of the pulse).
c) Derive a plot of the desired k_z versus time. Use this plot to design your gradient waveforms. Make sure to clearly label all amplitudes and timing parameters.
d) Draw a quiver diagram that shows the orientation of the spins from each RF pulse at the end of the pulse sequence. Discuss how the sum of the spin profiles achieves the desired overall profile. In specific, test out the locations where you expect the profile to equal 0, ½ or 1. Make sure to take into account the flip angles of the RF pulses that you used.

Problem 4

In this exercise you will experiment with how different imaging parameters alter the contrast of an image. First download the file BE280A08_hw6.mat from the course website. In this file you will find three matrices labeled csf, gm, and wm, which are the partial volume maps for cerebral spin fluid, gray matter, and white matter, respectively. Assume the values shown in the table below. Assume that your MRI system can have a minimum echo time (TE) of 3 ms and a maximum TR of 15,000ms. Finally, assume that you are using a saturation-recovery sequence. Come up with sequence parameters that yield proton-density, T1-weighted, and T2-weighted images and use the partial volume maps to generate corresponding images. For the T1-weighted image, choose parameters that maximize the contrast between gray and white matter -- you may want to use MATLAB to search over possible TR values.

Tissue	Proton Density	T1 (ms)	T2 (ms)
Csf	1.0	4000	2000
Gray	0.85	1350	110
White	0.7	850	80

Problem 5 Multiple Choice Problems

1) In an fMRI experiment, as the deoxyhemoglobin content increases in a vessel, the transverse relaxation rate R_2^* will
   a) decrease due to increased dephasing of the spins
   b) decrease due to decreased dephasing of the spins
   c) increase due to increased dephasing of the spins
   d) increase due to decreased dephasing of the spins
   e) remain unchanged

2) In a diffusion MRI experiment, as the diffusivity of the medium increases, the MR signal will
   a) increase due to increased dephasing of the spins
   b) increase due to decreased dephasing of the spins
   c) decrease due to increased dephasing of the spins
   d) decrease due to decreased dephasing of the spins
   e) remain unchanged.

3) You are asked to design a T_2 weighted pulse sequence. The optimal choice of parameters is
   a) Use a gradient echo pulse sequence with a short TR and short TE.
   b) Use a spin echo pulse sequence with a short TR and short TE.
   c) Use a gradient echo pulse sequence with a long TR and long TE.
   d) Use a spin echo pulse sequence with a long TR and long TE.
   e) Use a spin echo pulse sequence with a short TR and long TE.

4) You are considering purchasing a system that has 3 modules. The spatial resolutions (FWHM) of the various modules are 2 mm, 2 mm, and 3 mm. You may assume that the overall system response is obtained from the linear convolution of the individual module responses. The overall spatial resolution of the entire system is approximately.
   a) 1 mm
   b) 2 mm
   c) 3 mm
   d) 4 mm
   e) 5 mm

5) You have designed an RF pulse to excite a slice that is 5 mm thick with a slice select gradient of amplitude 1 G/cm. The user decides to reduce the slice thickness to 2.5 mm. The amplitude of the slice select gradient will now be:
   a) 0.5 G/cm
   b) 1.0 G/cm
   c) 1.5 G/cm
   d) 2.0 G/cm
   e) 2.5 G/cm